ISGSR 2011 - Vogt, Schuppener, Straub & Bräu (eds) - © 2011 Bundesanstalt für Wasserbau ISBN 978-3-939230-01-4

Shallow Foundation Design through Probabilistic and Deterministic Methods

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ABSTRACT: The design of a shallow foundation with eccentric loading is presented for the ultimate limit state of the bearing resistance, according to the formulation presented in annex D of NP EN 1997-1:2010. Probabilistic and deterministic methods were used. Concerning probabilistic methods, the approximate probabilistic methods, advanced first-order second-moment method (*AFOSM*) and first-order second-moment method (*FOSM*), were applied. For the deterministic calculation, the partial safety factors method recommended by the Eurocode and applied in most practical cases, was implemented. It was assumed that problem variables, such as loads (permanent and variable vertical loads) and soil parameters, follow normal distribution functions. However, the horizontal variable load and the depth of foundation were described by the Gumbel and the rectangular distribution functions, respectively. The results obtained by approximate probabilistic methods were validated by Monte Carlo simulations. Comparisons were made between the results of the three design methods used.

Keywords: Shallow foundation; Hasofer-Lind method; Bearing resistance; Partial safety factor; Probabilistic methods.

1 INTRODUCTION

The traditional approach used in structural analysis and design is deterministic. In these methods, the characteristic values of the random variables are usually considered. However, the respectively random variable uncertainties are indirectly taken into account via partial safety factors calibrated semi-probabilistically, which is essentially, according to Massih and Soubra (2008), in part, a "factor of ignorance", but also, to take into account design situation and parameters not considered in the analysis. As an alternative to the previous method, one can use probabilistic approaches, that are a more rational way of structural analysis and design, which enables to consider directly the inherent uncertainty of each variable in the problem under consideration.

The Eurocode design philosophy (NP EN 1990:2009) prescribes the partial safety factors method as the principal design method. However, the possibility of applying probability methods is also given.

The design of the width B of a square shallow foundation, subjected to an eccentric load, arising from the application of deterministic and probabilistic methods, is herein presented and compared for the ultimate limit state verification of the bearing resistance. As stated into Eurocode 7 (NP EN 1997-1:2010), *EC7*, the geotechnical structures design can be done by analytical, numerical, semi-empirical and prescriptive methods. The design methodology implemented in this paper belongs to the analytical group and follows the formulation presented in annex D of *EC7*. Therefore, in drained conditions and in a homogenous sandy soil with a near horizontal surface, the soil bearing resistance can be obtained by Eq. (1), formulated by the theory of plasticity and based on experimental results. In Eq. (1), N_q and N_γ are the soil bearing capacity factors, s_q and s_γ foundation shape factors, i_q and i_γ coefficients due to load inclination, q' the effective stress at the depth of foundation, γ' the effective soil unit weight, B' the effective width of the shallow foundation and R/A' is the ultimate vertical stress, with $A'=B \cdot B'$. The expressions of the previous variables can be found in annex D of *EC7*. Figure 1 represents a sketch of the problem under study.

$$\frac{R}{A'} = q' N_q s_q i_q + \frac{1}{2}\gamma' B' N_\gamma s_\gamma i_\gamma \quad (1)$$

In the next section, the random variables considered in the problem are introduced and characterized. The calculation of the width, B, of a square shallow foundation is done in the following for the ultimate limit state of the bearing resistance, through an approximate probabilistic method, developed by Hasofer-Lind (1974). The results obtained are then compared with Monte Carlo simulations. To conclude about the non-linearity of the problem and the applicability of the probabilistic simplified approaches, the same problem was solved using the mean value first-order second-moment (*MVFOSM*). Finally, the shallow foundation was designed based on *EC7*, with the partial safety factors method and some conclusions are drawn.

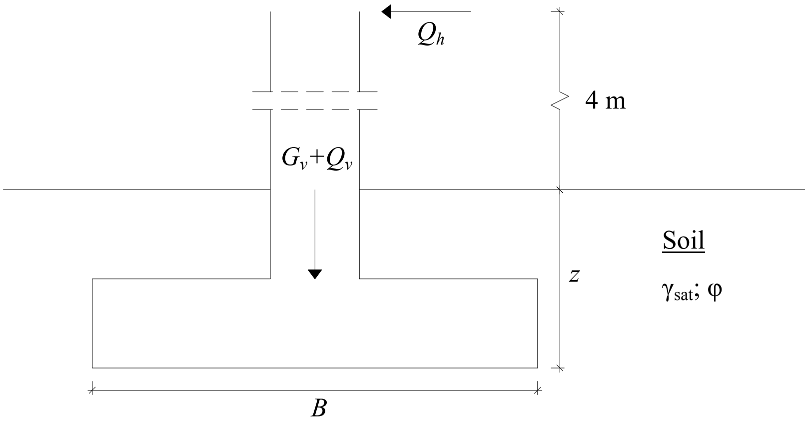

Figure 1. Sketch of the problem.

2 RANDOM VARIABLES

In the probabilistic design, the uncertainties of the loads in time, the soil properties in space and the depth of foundation were directly considered. In Table 1, the random variables considered in the problem are characterized.

Table 1. Random variables properties.

Random variable	Distribution function	μ	CV
Permanent vertical load	Normal	3000	0.10
Variable vertical load	Normal	1000	0.50
Variable horizontal load	Gumbel	250	0.25
Saturated soil weight	Normal	20	0.05
Soil friction angle	Normal	32	0.07
Depth of foundation	Rectangular between z = 1.5 and z = 2.5 m		

The uncertainty quantification of the actions was done according to the Joint Committee on Structural Safety recommendations, JCSS (2001). As a result, a normal distribution was selected to represent the permanent and variable vertical loads with a coefficient of variation, CV, of 10 and 50 %, respectively. JSCC considers the live load constituted by three parts: the overall mean load intensity for a particular user category, a zero mean normal distributed variable and a zero mean random field with a characteristic skewness to the right. For simplicity, only the first and second parts were considered, assuming a constant spatial distribution. Taking into account that the horizontal variable load was due to the wind action, the Gumbel distribution, with a CV of 25 %, was considered for the type of uncertainty involved in this kind of natural phenomenon.

There are numerous studies that characterize and quantify the uncertainties of the physical and mechanical properties of soils. Based on studies of other researchers, Chalermyanont and Benson (2005) reported that a normal distribution is suitable to describe the unit weight and internal friction angle of soils. According to Phoon and Kulhawy (1999), the unit weight and the angle of internal friction typically have values of CV between 3 and 10 % and between 5 and 11 %, respectively, conveniently weighted along the mobilized soil mass. In the present communication *CV* values equal to 5% and 7% were considered for the unit weight and the internal friction angle, respectively.

Table 2 represents the correlation matrix assumed between the random variables, after some reflection about the physical behaviour of the variables.

Table 2. Random variables correlation matrix.

Random variable	Saturated soil weight	Soil friction angle
Saturated soil weight	1.0	0.5
Soil friction angle	0.5	1.0

3 PROBABILISTIC METHODS

According to the *EC0*, the structural safety verification, for one particular reliability level, is done through the limit state concept. A limit state is the limit beyond which the structure does not satisfy the relevant design criteria. So, for each structural system, the relevant limit state must not be exceeded during the lifetime of the structure, for any design situation with probability of occurrence.

Reliability is the probability of a structure properly performing the functions for which it was designed over a given time. The structural reliability is normally evaluated using two measures, related by

$$\beta = \Phi^{-1}\left(P_f\right) \tag{2}$$

where β is the reliability index and P_f is the failure probability. Φ^{-1} represents the inverse of the cumulative distribution of a standard normal variable. For current structures, with an expected lifetime of 50 yr, the *EC0* sets a minimum reliability index of 3.8 for the ultimate limit states design, which corresponds to a P_f = 7.2×10^{-5}, concerning RC2 class and CC2 (medium consequence for loss of human lives and considerable economic, social or environmental consequences). It was assumed in this paper that the shallow foundation is a current structure.

In general, the failure probability can be determined using: accurate analytical integration, numerical integration methods, approximate analytical methods (like *FORM* methods) and simulation methods. The *FORM* methods include the first-order second-moments methods, *FOSM*, and the advanced first-order second-moment methods, *AFOSM*.

3.1 *Hasofer-Lind method*

In its original form, the Hasofer-Lind method, which belongs to *AFOSM*, is applicable to problems with uncorrelated normal random variables. The corresponding reliability index is defined as the minimum distance from the origin of the reduced coordinate system to the performance function, $g(X')$ and can be expressed as

$$\beta_{HL} = \sqrt{(x'^*)^T (x'^*)} \tag{3}$$

where (x'^*) is the point of the performance function closest to the origin in reduced coordinates, named calculation or design point. In this definition, the original coordinate system $X = (x_1, x_2, \ldots, x_n)$ is transformed into a reduced coordinate system $X' = (x'_1, x'_2, \ldots, x'_n)$ according to Eq. (4). Thus, the annulment of the performance function is made in the reduced coordinate system, $g(X') = 0$.

For nonlinear performance functions, the minimum distance calculation is an optimization problem, defined by β_{HL} minimization, with the constraint condition $g(x) = g(x') = 0$. This calculation procedure was implemented in the program Mathcad 14. According to Low and Tang (1997), it is possible to consider the correlation between random variables in the value of the reliability index by Eq. (5), where ρ^{-1} is the inverse matrix of correlation coefficients.

$$X'_i = \frac{X_i - \mu_{X_i}}{\sigma_{X_i}} \tag{4}$$

$$\beta_{HL} = \sqrt{(x'^*)^T \rho^{-1} (x'^*)} \tag{5}$$

For random variables with non-normal distributions, the Rackwitz and Fiessler (1976) method was used to transform the variables distribution into an equivalent normal distribution. The estimation of the equivalent normal distribution parameters, $\mu^N_{X_i}$ and $\sigma^N_{X_i}$, is performed by imposing equality of the cumulative distribution functions, *F*, and probability density functions, *f*, at the design point,

$X'^* = (x'^*_1, x'^*_2, ..., x'^*_n)$ of the non-normal variables and the equivalent normal variables. The parameters of equivalent distributions were determined by

$$\sigma^N_{X_i} = \frac{\phi\{\Phi^{-1}[F_{X_i}(x^*_i)]\}}{f_{X_i}(x^*_i)} \quad (6) \qquad \mu^N_{X_i} = x^*_i - \Phi^{-1}[F_{X_i}(x^*_i)]\sigma^N_{X_i} \quad (7)$$

The ultimate limit state verifications of bearing resistance took into account four load combinations, presented in Table 3. As the problem contains two variable actions, the *EC0* states that, for each combination, one of these actions shall be selected as principal action and the other, named accompanying action, shall be affected by the coefficient, to take into account the reduced probability of the action variables simultaneously reach extreme values. In the probabilistic approach, the *EC0* defines the ψ_0 value for normal distribution according Eq. (8), where V is the *CV* of the accompanying action for the reference period, T_1 the greatest basic period of combined variable actions and T the reference period (50 yr). It was considered that the basic period for the vertical and horizontal variable actions is 7 yr (typical for imposed loads on building floors) and 1 yr (associated to climate actions), respectively. It was found that T_1 = 7 yr.

Table 3. Load combinations.

I	$G_v+\psi_0 Q_v+Q_h$
II	$G_v+Q_v+\psi_0 Q_h$
III	G_v+Q_h
IV	G_v+Q_v (load without eccentricity)

$$\psi_0 = \frac{1+(0.28\beta - 0.7\,ln(T/T_1))V}{1+0.7\beta V} \quad (8)$$

Table 4 presents the values of B for each of the four action combinations, with the respective design points. It was determined B = 4.56 m. In this example, the design was determined by the actions combinations I and III.

Table 4. Width B obtained by the Hasofer-Lind method, with the respective design points.

Load combinations	B (m)	ψ_0	γ_{sat}^* (kN/m^3)	φ^* (º)	z^* (m)	G_v^* (kN)	Q_v^* (kN)	Q_h^* (kN)
I	**4.56**	0.36	17.63	25.14	1.70	3059.29	383.66	387.56
II	4.42	0.55	17.39	24.47	1.68	3151.18	1419.82	146.67
III	**4.56**	-	18.25	26.84	1.78	2906.23	-	522.47
IV	4.12	-	17.42	24.53	1.67	3178.60	1495.34	-

3.2 *Monte Carlo simulations*

The validation of the results obtained by the previous method was performed by conducting Monte Carlo simulations. Using the program Risk, the best fit distributions of the results of four Monte Carlo simulations were determined. A simulation was run for each load combination, and the corresponding failure probability and reliability index were evaluated. Each simulation contained the generation of 100 000 sets of random numbers. The adjustment of the distribution function to the Monte Carlo simulation results was made applying chi-square method.

Table 5. Results from the Monte Carlo simulations and validation of the results obtained by Hasofer-Lind method.

Load combinations	B (m)	Best distribution fit	Distribution parameters	P_f	β_{MC}	β_{MC}/β_{HL}
I	4.56	LogNomal	μ = 985.15 σ = 385.7 Shift = -231.35	1.32×10^{-4}	3.65	0.96
II	4.42	LogNormal	μ = 1097.3 σ = 437.97 Shift = -227.84	4.88×10^{-5}	3.90	1.03
III	4.56	LogNormal	μ = 956.47 σ = 375.57 Shift = -218.09	9.97×10^{-5}	3.72	0.98
IV	4.12	LogNormal	μ = 1220.9 σ = 485.29 Shift = -245.39	3.20×10^{-5}	4.00	**1.05**

Table 5 represents the values obtained by Monte Carlo simulations and Figure 2 presents graphically the same results. The results obtained by the Hasofer-Lind method have small deviations compared to those obtained by Monte Carlo simulations. In this case, a maximum deviation of 5 % was found in the reliability index, for the load combination IV. Based on these results, the calculation of width *B*, by the Hasofer-Lind method, is considered as valid.

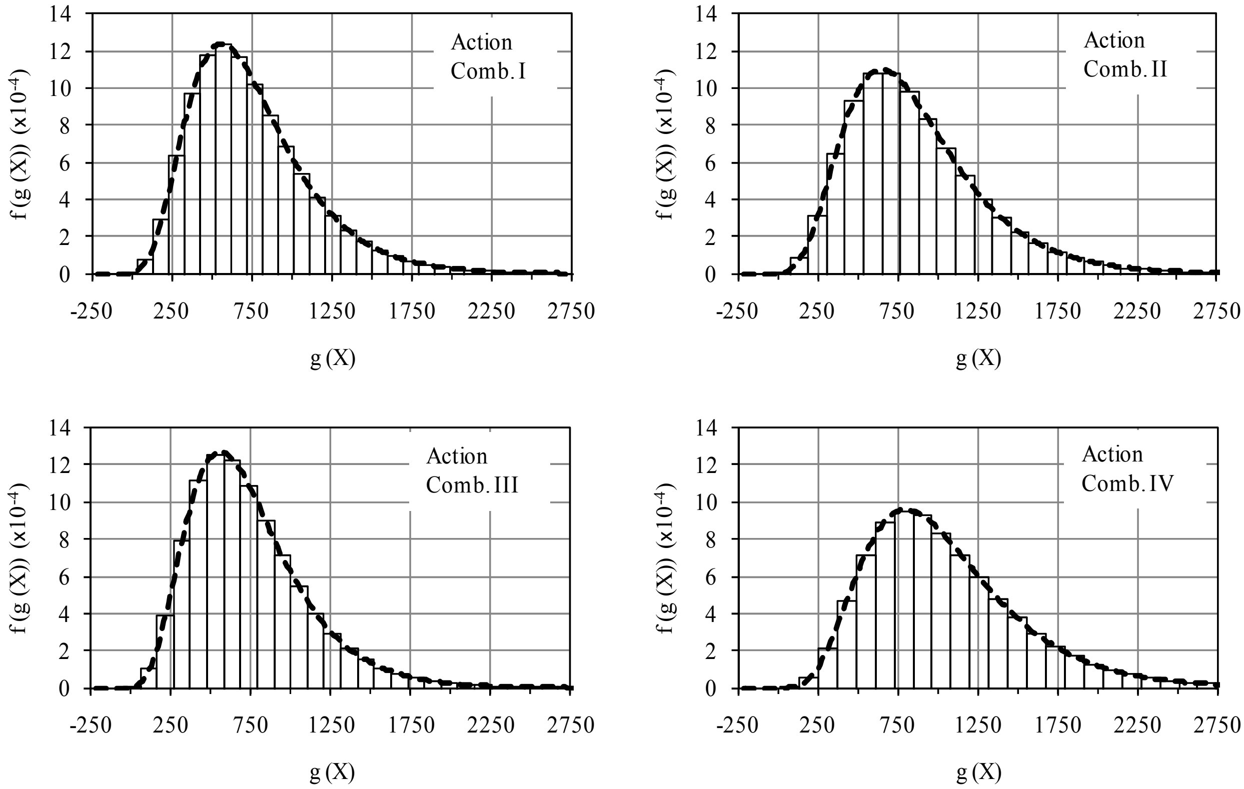

Figure 2. Monte Carlo simulation results.

3.3 *FOSM*

In this section, the *FOSM* was used to solve the same problem. *FOSM* is also known as the mean value first-order second-moment method, *MVFOSM*. In this method, the information of the random variables distribution is ignored. The performance function is linearized by the first-order approximation of a Taylor series development, evaluated at the mean values of the random variables, using the statistical moments up to the second order (mean values and variances). It comprises a higher degree of approximation than the Hasofer-Lind method.

Limiting the Taylor series expansion of the performance function to linear terms produces the expressions represented by Eq. (9) and Eq. (10), as first order approximation of the mean value and the variance, respectively.

$$\mu = g\left(\mu_{X_1}, \mu_{X_2}, ..., \mu_{X_n}\right) \tag{9}$$

$$\sigma = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial g}{\partial X_i}\frac{\partial g}{\partial X_j}Cov\left(X_i, X_j\right)} \tag{10}$$

where

$$Cov\left(X_i, X_j\right) = \rho_{i,j}\sigma_i\sigma_j \tag{11}$$

In situations of lack of an explicit performance function, such as the results from numerical models, the determination of σ_z can be performed by the central difference approximation for the calculation of the first derivative (finite difference method). According to this method, the expressions represented in Eq. (12) to Eq. (15) are considered.

$$\frac{\partial g}{\partial X_i} \cong \frac{Y_i^+ - Y_i^-}{2\sigma_{X_i}} \tag{12}$$

$$Y_i^+ = g\left[\mu_{X_1}, \mu_{X_2}, \ldots, \left(\mu_{X_i} + \sigma_{X_i}\right), \ldots, \mu_{X_n}\right] \tag{13}$$

$$Y_i^- = g\left[\mu_{X_1}, \mu_{X_2}, \ldots, \left(\mu_{X_i} - \sigma_{X_i}\right), \ldots, \mu_{X_n}\right] \tag{14}$$

$$\sigma \cong \sqrt{\sum_{i=1}^{n} \left(\frac{Y_i^+ - Y_i^-}{2\sigma_{X_i}}\right)^2 \sigma_{X_i}{}^2} = \sqrt{\sum_{i=1}^{n} \left(\frac{Y_i^+ - Y_i^-}{2}\right)^2} \tag{15}$$

Table 6 represents the reliability index, $\beta = \mu_z / \sigma_z$, and its failure probability, obtained by *FOSM* for the load combinations used in 3.1. The results of the exact derivative and central difference approximation are presented.

Table 6. Reliability index and failure probability determined by *FOSM*.

Action combina-tions	B (m)	Exact derivative					Central difference approximation				
		ψ_0	μ	σ	β_{FOSM}	$P_{f,FOSM}$	ψ_0	μ	σ	$\beta_{FOSM,a}$	$P_{f,FOSM,a}$
I	4.56	0.349	737.2	362.1	2.036	2.09×10^{-2}	0.348	737.2	367.9	2.004	2.26×10^{-2}
II	4.42	0.590	812.7	398.4	2.040	2.07×10^{-2}	0.589	812.5	405.0	2.006	2.24×10^{-2}
III	4.56	-	727.6	353.6	2.058	1.98×10^{-2}	-	727.6	359.2	2.026	2.14×10^{-2}
IV	4.12	-	894.2	433.8	2.062	1.96×10^{-2}	-	894.2	441.1	2.027	2.13×10^{-2}

The difference between the results of the variance of the performance function, obtained by exact derivative or by central difference approximation, is very small, despite of the very sharp shape and non-linearity nature of the performance function. The reliability indexes obtained are quite similar in both cases. Thus, in the inability to determine the exact partial derivatives of different variables, the central difference approximation allows, in a simple manner, the determination of similar results to the derivation of the exact function of performance.

However, the results obtained by *FOSM* differ greatly from the results obtained by the Hasofer-Lind method, which have been confirmed by Monte Carlo simulations. The *FOSM* is only accurate in special situations, such as when all variables are normal and statistically independent and the performance function is almost a linear combination of these variables, which is not the present case. The absence of the distribution functions of the variables information and the use of a linearized performance function around its mean point can lead to significant errors. In this case, as shown in Figure 3, a reliability index of 3.8 could not be achieved with this method for any load combinations, despite of the width value considered. In this case, this approximate method does not produce acceptable results.

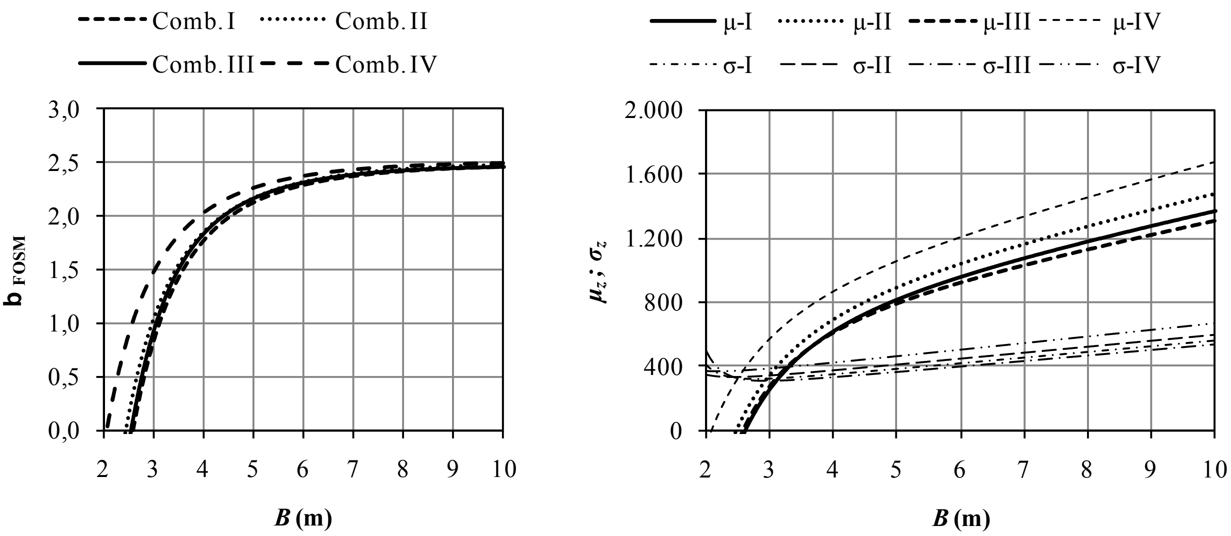

Figure 3. Reliability index, mean value and variance evolution with B for *FOSM*.

4 PARTIAL SAFETY FACTORS METHOD

According to the recommended in *EC7*, the design of the width B of a shallow foundation was made by the three approaches presented in Table 7, considering the partial safety factors presented in Table 8 and the characteristic values shown in Table 9, obtained from the distribution functions presented in 2.

This communication presents the comparison between the three Eurocode design approaches, even knowing that each European country adopted only one design approach.

Table 7. Eurocode design approaches.

Design approaches		Combinations
DA1	C1	A1 ”+” M1 ”+” R1
	C2	A2 ”+” M2 ”+” R1
DA2		A1 ”+” M1 ”+” R2
DA3		(A1 or A2) ”+” M1 ”+” R1

Table 8. Partial safety factors recommended by Eurocode.

Actions				Soil parameters		
Permanent	A1	fav	1	tan(φ)	M1	1
		un-fav	1.35		M2	1.25
	A2	fav	1	γ_{sat}	M1	1
		un-fav	1		M2	1
Variable	A1	fav	0	Resistance		
		un-fav	1.5	R1		1
	A2	fav	0	R2		1.4
		un-fav	1.3	R3		1

Table 9. Characteristic values obtained from the distribution functions of the random variables (*EC7*).

Random variables	Mean values	Characteristic values		Percentile (%)	Random variables	Mean values	Characteristic values		Percentile (%)
G_v (kN)	3000	3493.5	$G_{v,k,sup}$	95	γ_{sat} (kN/m^3)	20	21.65	$\gamma_{sat,k,sup}$	95
		2506.5	$G_{v,k,inf}$	5			18.36	$\gamma_{sat,k,inf}$	5
Q_v (kN)	1000	2026.9	$Q_{v,k,desf}$	98	φ (º)	32	35.7	$\varphi_{k,sup}$	95
		0	$Q_{v,k,fav}$	-			28.3	$\varphi_{k,inf}$	5
Q_h (kN)	250	412	$Q_{h,k,desf}$	98	z (m)	2	2.45	$z_{k,inf}$	95
		0	$Q_{h,k,fav}$	-			1.55	$z_{k,sup}$	5

Table 10 presents the results of width B for each design approach considered in Table 7. The corresponding design values of the six variables are also indicated. As can be seen in this example, for the DA1-C1 and DA2 approaches the horizontal load is most relevant while, in the other two approaches, the vertical loads increase their importance in comparison with the horizontal load effects.

In this case, the width B would be determined by the approach DA3, obtaining $B = 6.05$ m. This value is 33 % higher than the value obtained by the method of Hasofer-Lind. In the four approaches, the deterministic method gave always higher values of B than the probabilistic methods. Assuming, that the Eurocodes take into account the uncertainties considered in the probabilistic methods, this means that, for the variability assumed, the partial safety factors present in *EC7* were calibrated for lower failure probabilities. Table 10 represents the reliability indexes determined in accordance with the Hasofer-Lind method for the dimensions obtained by the partial safety factors method. As can be seen, all the values are higher than the limit imposed by *EC0* (3.8).

Table 10. Width B designed by the partial safety factors method, recommended by the *EC7*.

Calculation approaches		B_{EC7} (m)	$G_{v,d}$ (kN)	$Q_{v,d}$ (kN)	Q_h (kN)	γ_{sat} (kN/m^3)	φ (°)	z (m)	B_{EC7}/B_{HL}	β_{HL}
DA1	C1	4.93	2506.5	0	618.0	18.36	28.32	1.55	1.08	**4.23**
	C2	5.62	3493.5	1843.7*	535.6	18.36	23.32	1.55	1.23	**4.88**
DA2		5.41	2506.5	0	618.0	18.36	28.32	1.55	1.19	**4.70**
DA3		**6.05**	4716.2	2128.2*	618.0	18.36	23.32	1.55	1.33	**5.21**

* value affected by ψ_0

The Eurocodes partial safety factors were calibrated semi-probabilistically, taking into account the past relevant geotechnical experience, in order to not cause any design disruption. For comparison, Table 11 presents the partial safety factors determined from the results obtained in 2.1 with the probabilistic method. The results are very different from those proposed by *EC7*. In general, the partial factors are smaller than those recommended in the regulation. For the material properties, the values of the partial safety factors are close to those recommended by *EC7*. The values for the foundation level are near the unit, comparing with a characteristic value. This means that the consideration of the mean value for geometric variables with significant variance, recommended by *EC0*, is not the best option. At last, the loading partial safety factors are, in some cases, very different than the values suggested by the *EC0*.

However, in these analyses, the experience of historical cases was overlooked, which have in consideration another type of uncertainties, namely, spatial variability, construction activities and calculation model accuracy.

Table 11. Determination of the partial safety factors from the results obtained by the Hasofer-Lind method.

Action combinations	γ_γ	γ_φ	γ_z	G_v^* (kN)	Q_v^* (kN)	Q_h^* (kN)
I	1.041	1.147	$0.912(z_{k,sup})$	$0.876(G_{v,k,sup})$	$0.526(Q_{v,k,desf})$	$0.941(Q_{h,k,desf})$
II	1.056	1.183	$0.923(z_{k,sup})$	$0.902(G_{v,k,sup})$	$0.700(Q_{v,k,desf})$	$0.647(Q_{h,k,desf})$
III	1.006	1.064	$0.871(z_{k,sup})$	$0.832(G_{v,k,sup})$	-	$1.268(Q_{h,k,desf})$
IV	1.054	1.180	$0.928(z_{k,sup})$	$0.910(G_{v,k,sup})$	$0.738(Q_{v,k,desf})$	-

5 CONCLUSIONS

This paper presents the design of the width B of a square shallow foundation subjected to eccentric loading, through deterministic and probabilistic methods for the ultimate limit state of the bearing resistance. Width B was obtained by the Hasofer-Lind method. Those results were validated by Monte Carlo simulations and compared with other design methods, namely probabilistic and deterministic methods.

The *MVFOSM* utilization does not give adequate results, achieving undervalued levels of safety. This method should not be used in problems with high nonlinear solutions.

The results show that the level of safety determined by Hasofer-Lind method is smaller in comparison with the partial safety factors method. In that way, there are two possibilities to explain the differences: the partial safety factors method is overly conservative or the Hasofer-Lind method does not consider all the uncertainties of the problem.

The Eurocodes partial safety factors were calibrated semi-probabilistically by performing probabilistic calculations, being adjusted accordingly to the experience gained over time. What is shown by the results is that the direct utilization of probabilistic methods does not take into consideration important uncertainties related to construction activities and soil variability. So, without consideration of these types of uncertainties, these probabilistic methods should be applied carefully, due to the fact that can produce unsafe designs compared with the level of safety considered over time.

As future development of the present work, the different sources of uncertainties, namely, spatial variability, construction activities and calculation model precision, shall be incorporated in the probabilistic methods for designing shallow foundations, in order to incorporate the calibration of the partial safety factors or to establish the probabilistic methods as an alternative design methodology.

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