ICHE 2014, Hamburg - Lehfeldt & Kopmann (eds) - © 2014 Bundesanstalt für Wasserbau ISBN 978-3-939230-32-8

Propagation of Distant Tsunami across East Sea

J.-H. Lee, K. Rehman, H.-R. Cho & Y.-S. Cho
Department of Civil and Environmental Engineering, Hanyang University, Seoul, Korea

ABSTRACT: A second-order upwind scheme is employed to predict the run-up heights of tsunamis in the East Sea and the obtained results are compared with the available field observed data and numerical results of a first order upwind scheme. In the numerical model, the governing equations solved by the finite difference scheme are the linear shallow-water equations in deep water and the nonlinear shallow-water equations in shallow water. The target event is the 1983 Central East Sea Tsunami recorded as the most devastating tsunami in Korea. The predicted maximum run-up heights agree reasonably well with field observations.

Keywords: Tsunami, Shallow water equation, Second-order upwind scheme, Run-up height

1 INTRODUCTION

Tsunamis are large water waves set in motion either by landslides, submarine volcanic explosions, or sea bottom deformations associated with large submarine earthquakes. The West Asia Tsunami occurred on December 26, 2004 killed approximately 300,000 people and deprived of property of 10billion USD. The Tsunami was triggered by an 9.3-Mw magnitude earthquake at 3.316°N, 95.854°E off the coast of Sumatra in the Indonesian Archipelago at 06:29 hr, making it the most powerful in the world in the last 40 years. The earthquake epicenter was located at a relatively shallow depth, about 10 km below the ocean floor. The high magnitude (Mw 9.3) of the earthquake along with its shallow epicenter triggered tsunamis in the northeast Indian Ocean.

If an earthquake is generated, the displacement of the initial free surface of approximate 3-5m is generated. The displacement is then propagated to all directions at the velocity of by gravitational force, where h is a local water depth. In general, tsunami is analyzed by the linear theory because it has a very small wave height comparing with a depth in deep water. If the tsunamis approach at a shore, however the height of a wave increases by shoaling effects. Finally, the tsunamis not only kill many human beings but also cause serious property damages along the shoreline as they reach coastal area.

Pelinovsky et al. (1999) studied run-up of tsunamis on a vertical wall in a bay of different cross sections by using the shallow-water equations. However, the run-up heights should be evaluated on an inclined wall to make an inundation map. The solitary wave adopted in this paper is considered a valid representation of a tsunami (Silva et al., 2000) to present a reflection and transmission of tsunami waves by coastal structures.

2 GOVERNING EQUATIONS

In this section, the governing equations describing the propagation of tsunamis are first presented. The case of a constant water depth and a relatively small computational domain is chosen without considering the Coriolis effects. The linear Boussinesq equations over a constant depth may be written as (Mei, 1989).

$$\frac{\partial^2\zeta}{\partial t^2} - gh\left(\frac{\partial^2\zeta}{\partial x^2} + \frac{\partial^2\zeta}{\partial y^2}\right) = \frac{\partial h^3}{3}\left(\frac{\partial^4\zeta}{\partial x^4} + 2\frac{\partial^4\zeta}{\partial x^2\partial y^2} + \frac{\partial^4\zeta}{\partial y^4}\right) \quad (1)$$

in which ς is the free surface displacement, h is the still water depth and x and y are horizontal coordinates. However, the frequency dispersion terms of Eq. (1) have higher order derivatives, the accuracy of any numerical solutions depends on the technique used to approximate these terms and more computing time is needed for more accurate solutions. Therefore, solving the linear Boussinesq equations directly may not be an economical approach for simulating the transoceanic propagation of tsunamis (Imamura *et al*., 1988).

By denoting $P = uh$ and $Q = vh$ as the volume flux components in the x- and y-directions, respectively, the linear shallow-water equations are written as

$$\frac{\partial\zeta}{\partial t} + \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} = 0 \quad (2)$$

$$\frac{\partial P}{\partial t} + gh\frac{\partial\zeta}{\partial x} = 0 \quad (3)$$

$$\frac{\partial Q}{\partial t} + gh\frac{\partial\zeta}{\partial y} = 0 \quad (4)$$

In this study, the scheme proposed by Cho and Yoon (1998) is used to make the numerical dispersion equal to the physical dispersion. They proposed a leap-frog finite difference scheme as follows:

$$\frac{\zeta_{i,j}^{n+\frac{1}{2}} - \zeta_{i,j}^{n-\frac{1}{2}}}{\Delta t} + \frac{P_{i+\frac{1}{2},j}^{n} - P_{i-\frac{1}{2},j}^{n}}{\Delta x} + \frac{Q_{i-\frac{1}{2},j}^{n} - Q_{i-\frac{1}{2},j}^{n}}{\Delta y} = 0 \quad (5)$$

$$\frac{P_{i+\frac{1}{2},j}^{n+1} - P_{i-\frac{1}{2},j}^{n}}{\Delta t} + gh_{i+\frac{1}{2},j}\frac{\zeta_{i+1,j}^{n+\frac{1}{2}} - \zeta_{i,j}^{n+\frac{1}{2}}}{\Delta x}$$
$$+ \frac{\gamma gh}{12\Delta x}\left[\left(\zeta_{i+1,j+1}^{n+\frac{1}{2}} - 2\zeta_{i+1,j}^{n+\frac{1}{2}} + \zeta_{i+1,j-1}^{n+\frac{1}{2}}\right) - \left(\zeta_{i,j+1}^{n+\frac{1}{2}} - 2\zeta_{i,j}^{n+\frac{1}{2}} + \zeta_{i,j-1}^{n+\frac{1}{2}}\right)\right] = 0 \quad (6)$$

$$\frac{Q_{i,j+\frac{1}{2}}^{n+1} - Q_{i,j+\frac{1}{2}}^{n}}{\Delta t} + gh_{i,j+\frac{1}{2}}\frac{\zeta_{i,j+1}^{n+\frac{1}{2}} - \zeta_{i,j}^{n+\frac{1}{2}}}{\Delta y}$$
$$+ \frac{\gamma gh}{12\Delta y}\left[\left(\zeta_{i+1,j+1}^{n+\frac{1}{2}} - 2\zeta_{i,j+1}^{n+\frac{1}{2}} + \zeta_{i-1,j+1}^{n+\frac{1}{2}}\right) - \left(\zeta_{i+1,j}^{n+\frac{1}{2}} - 2\zeta_{i,j}^{n+\frac{1}{2}} + \zeta_{i-1,j}^{n+\frac{1}{2}}\right)\right] = 0 \quad (7)$$

in Eqs. (6) and (7) γ takes a value of one for the proposed scheme, while γ takes a value of zero for the Imamura and Goto's scheme(1988).

If Δt and Δx are chosen according to the relationship suggested by Imamura and Goto, the numerical dispersion generated from the modified scheme given by Eq. (5)-(7) mimics the frequency dispersion of the linear Boussinesq equations. Furthermore, the accuracy of the numerical scheme has been raised from a second order to a third order. A staggered grid system is employed in this study.

As a tsunami approaches a coastal area, the wave length of the incident tsunami becomes shorter and the amplitude becomes larger as the leading wave of a tsunami propagates into shallower water. Therefore, the nonlinear convective inertia force and bottom friction terms become increasingly important. The nonlinear shallow-water equations including bottom frictional effects are adequate to describe the flow motion in the coastal zone (Kajiura and Shuto, 1990; Liu et al., 1994). The nonlinear shallow-water equations are given as follows:

$$\frac{\partial\zeta}{\partial t} + \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} = 0 \quad (8)$$

$$\frac{\partial P}{\partial t} + \frac{\partial}{\partial x}\left(\frac{P^2}{H}\right) + \frac{\partial}{\partial y}\left(\frac{PQ}{H}\right) + \mathrm{g}H\frac{\partial\zeta}{\partial x} + \tau_x H = 0 \quad (9)$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{PQ}{H}\right) + \frac{\partial}{\partial y}\left(\frac{Q^2}{H}\right) + \mathrm{g}H\frac{\partial\zeta}{\partial y} + \tau_y H = 0 \quad (10)$$

where ζ is the free surface displacement, P is the horizontal components of the volume flux in the x-direction, Q, the horizontal components of the volume flux in the y-direction and H, the total water depth (still water depth + free surface displacement). Bottom friction terms can be modeled by using Manning's formula $\tau_x = \frac{gn^2}{H^{10/3}} P(P^2 + Q^2)^{1/2}$, $\tau_y = \frac{gn^2}{H^{10/3}} Q(P^2 + Q^2)^{1/2}$,where n is the Manning's relative roughness coefficient.

In previous studies, a first-order upwind scheme is widely and plausibly used to calculate the nonlinear terms of the momentum equations for analyzing a tsunami. In this study, however, a second-order upwind scheme is used to improve accuracy of analysis of the nonlinear terms in the momentum equations.

The second-order upwind scheme proposed by Shyy(1985) can be written as:

$$\frac{\partial(u\phi)}{\partial x_{i,j}} =$$

$$\frac{1}{2\Delta x}\left[3(u\phi)_{i,j} - 4(u\phi)_{i-1,j} + (u\phi)_{i-2,j}\right], \quad (u \geq 0)$$

$$\frac{1}{2\Delta x}\left[-3(u\phi)_{i,j} + 4(u\phi)_{i+1,j} - (u\phi)_{i+2,j}\right], \quad (u < 0) \tag{11}$$

The nonlinear terms of Eqs. (9) and (10) can be discritized by a second-order upwind scheme as followings:

$$\frac{\partial}{\partial x}\left(\frac{P^2}{H}\right) =$$

$$\frac{1}{2\Delta x}\left[3\frac{(P^2)^n_{i+1/2,j}}{H^n_{i+1/2,j}} - 4\frac{(P^2)^n_{i-1/2,j}}{H^n_{i-1/2,j}} + \frac{(P^2)^n_{i-3/2,j}}{H^n_{i-3/2,j}}\right], \quad P^n_{i+1/2,j} \geq 0$$

$$\frac{1}{2\Delta x}\left[-3\frac{(P^2)^n_{i+1/2,j}}{H^n_{i+1/2,j}} + 4\frac{(P^2)^n_{i+3/2,j}}{H^n_{i+3/2,j}} - \frac{(P^2)^n_{i+5/2,j}}{H^n_{i+5/2,j}}\right], \quad P^n_{i+1/2,j} < 0 \tag{12}$$

$$\frac{\partial}{\partial y}\left(\frac{PQ}{H}\right) =$$

$$\frac{1}{2\Delta y}\left[3\frac{(PQ)^n_{i+1/2,j}}{H^n_{i+1/2,j}} - 4\frac{(PQ)^n_{i+1/2,j-1}}{H^n_{i+1/2,j-1}} + \frac{(PQ)^n_{i+1/2,j-2}}{H^n_{i+1/2,j-2}}\right], \quad Q^n_{i+1/2,j} \geq 0$$

$$\frac{1}{2\Delta y}\left[-3\frac{(PQ)^n_{i+1/2,j}}{H^n_{i+1/2,j}} + 4\frac{(PQ)^n_{i+1/2,j+1}}{H^n_{i+1/2,j+1}} - \frac{(PQ)^n_{i+5/2,j+2}}{H^n_{i+5/2,j+2}}\right] \quad Q^n_{i+1/2,j} < 0 \tag{13}$$

$$\frac{\partial}{\partial x}\left(\frac{PQ}{H}\right) =$$

$$\frac{1}{2\Delta x}\left[3\frac{(PQ)^n_{i,j+1/2}}{H^n_{i,j+1/2}} - 4\frac{(PQ)^n_{i-1,j+1/2}}{H^n_{i-1,j+1/2}} + \frac{(P^2)^n_{i-2,j+1/2}}{H^n_{i-2,j+1/2}}\right], \quad P^n_{i,j+1/2} \geq 0$$

$$\frac{1}{2\Delta x}\left[-3\frac{(PQ)^n_{i,j+1/2}}{H^n_{i,j+1/2}} + 4\frac{(PQ)^n_{i+1,j+1/2}}{H^n_{i+1,j+1/2}} + \frac{(PQ)^n_{i+2,j+1/2}}{H^n_{i+2,j+1/2}}\right], \quad P^n_{i,j+1/2} < 0 \tag{14}$$

$$\frac{\partial}{\partial y}\left(\frac{Q^2}{H}\right) =$$

$$\frac{1}{2\Delta y}\left[3\frac{(Q^2)^n_{i,j+1/2}}{H^n_{i,j+1/2}} - 4\frac{(Q^2)^n_{i,j-1/2}}{H^n_{i,j-1/2}} + \frac{(Q^2)^n_{i,j-3/2}}{H^n_{i,j-3/2}}\right], \quad Q^n_{i,j+1/2} \geq 0$$

$$\frac{1}{2\Delta y}\left[-3\frac{(Q^2)^n_{i,j+1/2}}{H^n_{i,j+1/2}} + 4\frac{(Q^2)^n_{i,j+3/2}}{H^n_{i,j+3/2}} - \frac{(Q^2)^n_{i,j+5/2}}{H^n_{i,j+5/2}}\right], \quad Q^n_{i,j+1/2} < 0 \tag{15}$$

3 NESTING SCHEME

A moving boundary treatment is employed along the coastline of the Korean Peninsula to estimate maximum run-up heights of tsunamis. As shown in Figure 1, the East Sea surrounded by Korea, Japan and Russia has a specific bottom topography. The calculating time is too long in the East Sea because Domain O is consisted of 120,001×120,001 grids with a uniform size of 10m. Thus, the numerical analysis of tsunamis in the East Sea is done with various grid systems. The coarsest grid is 1.11km in deep water, whereas the finest is 4.5m near the shoreline.

Although the model can be run in the spherical coordinate system, the model was run only in the Cartesian coordinate system because effects of the spherical coordinate are negligible (Lee, 1999). Figure 1 and Figure 2 represent the linear and nonlinear domain of analysis of tsunamis.

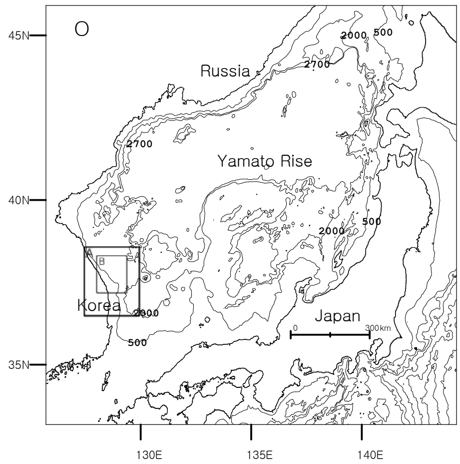

Figure *1*. *The linear domain of analysis of tsunamis*

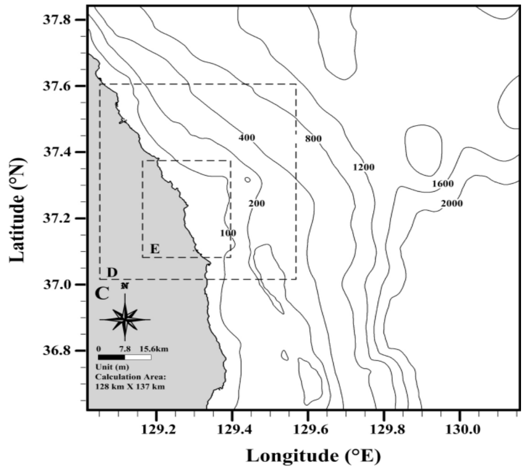

Figure 2. The nonlinear domain of analysis of tsunamis

4 NUMERICAL SIMULATIONS

4.1 *1983 Central East Sea Tsunami*

The 1983 Central East Sea Tsunami was occurred on May 26, 1983 at the nearshore of Akida, Japan. The Richter scale of the tsunami was 7.7. Figure 3 presents the occurrence location of the tsunami. The parameters of the earthquake are listed in Table 1.

Table 1. Parameters of Central East Sea Earthquake (Aida, 1984)

Description	N(^oN)	E(^oE)	H(km)	θ(^{o})	δ(^{o})	λ(^{o})	L(km)	W(km)	u(km)
Fault 1	40.21	138.84	2	22	40	90	40	30	760
Fault 2	40.54	139.02	3	355	25	80	60	30	305

Figure 3. The occurrence location of 1983 Tsunami

First, the tsunami simulated at the east coast by coarse grid. The results are presented by Figure 4 which is compared observed with present study. The computed results represent reasonably well the wave heights of the tsunami at the east coast. Especially, the wave heights are great near Imwon because of lens effects due to the Yamato Rise.

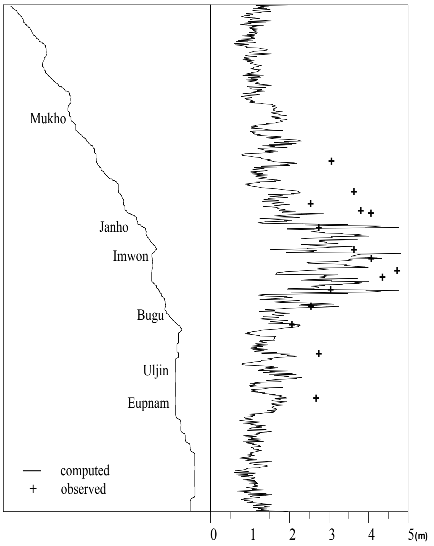

Figure 4. Wave heights of the tsunami at east coast for 1983 tsunami

The predicted run-up heights at 4 locations of the East Coast are compared with field observation and those of Lee in Table 2. The present study is more accurate than Lee's results.

Table 2. The comparison of the results of the east coast for 1983 tsunami

Description	Janho	Imwon	Bugu	Eupnam
Oberved	2.9	3.7	2.5	2.6
Lee	2.7	4.3	2.2	1.8
Present	2.7	4.2	2.6	2.3

Imwon is simulated by fine grid, $\Delta x = 4.5m$. This results are represented by Figure 5. The maximum run-up height is 4.8m.

The run-up heights compared with observed result. The result of simulation is similar to observed result at Geobuk market and breakwaters.

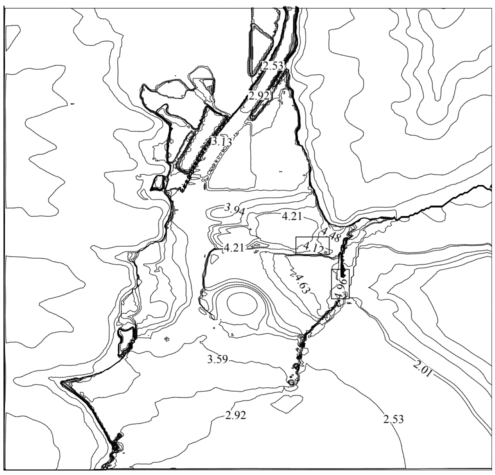

Figure 5. Run-up heights at Imwon for 1983 tsunami

Table 3. Comparison of run-up heights at Imwon for 1983 tsunami

Description	Geobuk market	breakwaters
Observed	3.6	5.0
Present	4.0	5.1

5 CONCLUDING REMARKS

In this study, the numerical model is based on the shallow-water equations, which developed by Shyy(1985) has been applied run-up at the east coast. The target events is 1983 tsunami. The results have been compared with observed data and that of first-order upwind scheme. The results are simulated at the east coast are more accurate than that of first-order upwind scheme. And, the tsunamis are simulated at Imwon by $\Delta x = 4.5m$. The results are greater than observed data, but the results are more accurate that first-order scheme. Thus, the computed results represent reasonably well the run-up heights of the tsunami at the east coast.

We can use the results to minimize loss of human lives and property damage from unexpected tsunami attacks, it is essential to construct a safety zone along the coastline based on the maximum inundation map developed by simulating historical and probable maximum tsunami events with this numerical model.

6 ACKNOWLEDGEMENTS

This research was supported by a grant [NEMA-ETH-2012-5] from the Earthquake and Tsunami Hazard Mitigation Research Group, National Emergency Management Agency of Korea and by the Research Program through the National Research Foundation of Korea funded by the Ministry of Education, Science and Technology (No.2011-0015386).

REFERENCES

Cho, Y.-S., (1995). Numerical simulations of tsunami propagation and run-up. Ph.D. thesis, Cornell University.

Cho, Y.-S. and Liu, P.L.-F., (1999). Crest length effects in nearshore tsunami run-up around islands. J. Geophys. Res. 104, 7907-7913.

Cho, Y.-S. and Yoon, S.-B., (1998). A modified leap-frog scheme for linear shallow-water equations. Coastal Engineering Journal, 40, 2, 191-205.

Kajiura, K. and Shuto, N., (1990). "Tsunami," in The SEA. edited by B. Le Mehaute, and D.M. Hanes, 395-420, John Wiley, New York.

Lee, H.-J., Imamura, F., Shuto, N., (1999). Characteristics of Tsunami Behaviors in the East sea. KSCE, Vol. 19, No. 2-3, pp. 401,409.

Liu, P.L.-F., Cho, Y.-S., Yoon, S.B. and Seo, S.N., (1994). Numerical simulations of the 1960 Chilean tsunami propagation and inundation at Hilo, Hawaii. in Recent Development in Tsunami Research, edited by M.I. El-Sabh, Kluwer Academic Publishers

Liu, P.L.-F., Cho, Y.-S., Briggs, M.J., Kanoglu, U. and Synolakis, C.E., (1995). Run-up of solitary wave on a circular island. J. Fluid Mech. 302, 259-285.

Yoon, S.-B., and Cho, J.-H., (2001). Numerical simulation of coastal inundation over discontinuous topography. Water Engineering Research, KWRA, 2, 2, 75-87.

Pelinovsky, E., Troshina, E., Golinko, V., Osipenko, N. and Petrukhin, N., (1999). Runup of tsunami waves on a vertical wall in a basin of complex topography. Phys. Chem. Earth (B). 214(5), 431-436.

Silva, R., Losada, I.J. and Losada, M.A., 2000. Reflection and transmission of tsunami waves by coastal structures. Appl. Ocean Res. 22, 215-223