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Calculation of Sediment Transport Capacity of Flowing Water in Rivers with Machine Learning

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ABSTRACT: Fluvial sediment transport literature is characterized by an abundance of studies comparing sediment transport formulae, which calculate either the bed-material load (mainly for sand-bed rivers) or the bed load (mainly for gravel-bed rivers). However, these studies exhibit different results and conclusions, exposing the insufficiency of the sediment transport formulae utilized for engineering projects. The present paper shows the potential of machine learning in quantifying fluvial sediment transport by summarizing the work done by the authors in the recent past and complementing this review by presenting new results from the support vector regression implementation. The generated models are robust and the results are encouraging, given the complexity of the problem and the inevitable noise inclusion from the field measurements, and superior to those of some of the well-known sediment transport formulae. The conclusions of this study support that the regression scheme is of importance, since all the input combinations tested (based mainly on shear stress, or stream power, or unit stream power) generated similarly good results, with respect to the machine learning technique employed, without having to set a threshold for the initiation of motion, thus avoiding erroneous zero transport rate predictions.

Keywords: Artificial neural networks (ANNs), Adaptive-network-based fuzzy inference system (ANFIS), Bed load, Bed-material load, Machine learning, Sediment transport, Support vector regression (SVR), Symbolic regression (SR)

1 INTRODUCTION

Sediment transport rate in streams and rivers is very difficult to be quantified and it constitutes a problem that challenges engineers, geomorphologists, and river scientists, since it is a prerequisite for numerous studies such as river restoration, habitat maintenance, dam design, etc. Many sediment transport functions have been proposed during the last years by several researchers based on different concepts and approaches; however, the generated results are, usually, in disagreement and may derive miscalculations of several orders of magnitude (Gomez and Church, 1989; Yang, 2003). Subsequently, a deterministic model that obeys the laws of physics and could be potentially used with confidence remains elusive due to the augmented complexity of the phenomenon and the large input data requirements.

Machine learning can provide solutions to problems where the actual knowledge of the physics and the internal mechanisms of the problem are not fully understood, and given a good quality dataset that encompasses an adequate part of the parameter space, can eventually generate formulae that can be used for future predictions. Witten et al. (2011) argued that certain classes of model syntax may be inappropriate as a representation of a physical system and, subsequently, no single machine learning technique is appropriate to all data mining problems. Consequently, in this paper, four different machine learning techniques are implemented and their results are compared for their efficacy with some well-known sediment transport functions from the literature. The utilized techniques are Artificial Neural Networks (ANNs), Adaptive-Network-Based Fuzzy Inference System (ANFIS), Symbolic Regression (SR) based on Genetic Programming, and Support Vector Regression (SVR). In addition, three different input combinations based mainly on variables highly correlated with sediment transport, such as unit stream power, stream power, and shear stress, are compared, without the need to set a criterion for the initiation of movement.

This paper concerns bed-material load and bed load prediction in sand-bed and gravel-bed rivers, respectively, and summarizes the work done by the authors in the recent past (Kitsikoudis et al., 2014a; 2014b), complemented by new results from the SVR implementation. The results are encouraging, given the complexity of the phenomenon and the noise that is implanted to the field measurements. Moreover, it is shown that all the input combinations generate similarly good outputs, with respect to the data-driven technique employed, accentuating the importance of the regression model.

2 SEDIMENT TRANSPORT

Sediment transport may exhibit several peculiarities, depending on the river under consideration. In sand-bed rivers, the majority of the sediment load is transported as suspended load, which may either amplify or damp turbulence, hence the flow resistance, depending on the relative magnitude of flow and sediment transport variables (Squires and Eaton, 1990). In gravel-bed rivers, due to the coarser grain size, a significant portion of the sediment load moves as bed load, which is of major importance for geomorphologic reasons. In addition to sorting by grain size across and along the streambed surface, gravel-beds tend to also exhibit vertical sorting, wherein the surface of the streambed is coarser than the underlying, subsurface, material (Parker and Sutherland, 1990). This leads to different transport phases, depending on the intensity of the flow (Recking, 2010).

Parker and Anderson (1977) expressed the sediment transport rate as a function of five dimensionless variables and derived the following relationship for equilibrium flow in an alluvial channel with a bed comprising non-cohesive sediment:

$$C = f\left(\hat{X}_1, \hat{X}_2, Re_{p50}, R, \sigma\right) \tag{1}$$

where C denotes a dimensionless variable expressing the sediment transport rate, $\hat{X}_1$ and $\hat{X}_2$ are dimensionless parameters closely tied to sediment transport, σ is the arithmetic standard deviation of the stream bed grain size distribution, Re_{p50} denotes an explicit particle Reynolds number, and R is the submerged specific gravity of the sediment, which is usually 1.65 for the most common natural sediments in rivers, and consequently it is omitted. The usage of σ in sand-bed rivers can be redundant (Kitsikoudis et al., 2014a) due to the surface grain size relative uniformity. After an extensive trial-and-error procedure, the $\hat{X}_1$ variable is replaced by Froude number, while for the $\hat{X}_2$ variable, three dimensionless variables based on unit stream power VS, stream power ω, and shear stress τ, which are highly correlated with sediment transport, will be tested in order to additionally facilitate a straightforward comparison of these variables, since the regression models will be trained on the same data. Yang (1973) made unit stream power dimensionless by dividing the product VS to fall velocity. However, for the gravel-bed streams, the dimensionless variable used is the one shown in Eq. (5), because fall velocity calculation in poorly sorted gravel-bed can be further problematic. The variables employed are given in the following equations.

particle Reynolds number: $Re_{p50} = \frac{\sqrt{Rgd_{50}}d_{50}}{v}$ (2)

Froude number: $Fr = \frac{V}{\sqrt{gD}}$ (3)

dimensionless unit stream power $= \frac{VS}{\omega_s}$ (4)

dimensionless unit stream power utilized for gravel-bed rivers $= \frac{VS}{\sqrt{gd_{50}}}$ (5)

shear Reynolds number: $Re^* = \frac{U_* d_{50}}{v}$ (6)

dimensionless stream power: $\omega^* = \frac{\omega}{\rho[g(\rho_s/\rho-1)d_{50}]^{3/2}}$ (7)

dimensionless shear stress or Shields number: $\tau^* = \frac{\tau}{g(\rho_s-\rho)d_{50}}$ (8)

where, g is the gravitational acceleration, d_{50} is the median grain diameter, v is the water kinematic viscosity, V is the mean flow velocity, D is the mean flow depth, the product VS denotes unit stream power, ω_s is the particle fall velocity, $U_* = \sqrt{gR_hS}$ is the shear velocity, R_h is the hydraulic radius, S is the energy slope, $\omega = \tau V$ is the stream power, ρ is the water density, ρ_s is the sediment density, and $\tau = \rho U_*^2$ is the shear stress. Table 1 shows the input combinations utilized for each case.

No threshold for the initiation of motion has been assigned to the input variables, since at flows of low intensity, the possibility of movement becomes very small but never equals zero due to turbulence fluctuations (Lavelle and Mofjeld, 1987). As a result, there are no erroneous zero transport rate predictions.

Table 1. Input combinations

	Sand-bed rivers	Gravel-bed rivers
(a)	$Re_{p50}^{2/3}$, Fr, Re^*, VS/ω_s	Re_{p50}, σ, Fr, Re^*, $VS/(gd_{50})^{1/2}$
(b)	$Re_{p50}^{2/3}$, Fr, ω^*	Re_{p50}, σ, Fr, ω^*
(c)	$Re_{p50}^{2/3}$, Fr, τ^*	Re_{p50} σ, Fr, τ^*

3 MACHINE LEARNING

The recorded observations of a system can be further analyzed in the search for the information they encode. Data mining aims at providing tools to facilitate the conversion of data into a number of forms, such as equations, which can provide a better understanding of the process generating or producing these data, and combined with the already available understanding of the physical processes result in improved predictive capability (Babovic, 2000). The data are usually divided into three sets. The training set trains the model on the basis of a minimization criterion, which is usually a sum of errors between the computed outputs and the actual measured data, and the validation set is used as a training stopping criterion to avoid overfitting to the data. The testing set is used to evaluate the generated model and assess its generalization capability. Since the scope of this paper is to exhibit the results of the machine learning implementation, and due to the space limitations, no theoretical information of the implemented techniques is provided herein. The methodology followed for the ANNs, ANFIS, and SR is described in Kitsikoudis et al. (2014a), along with the results for the bed-material load prediction in sand-bed rivers. The results for the bed load prediction in gravel-bed rivers are obtained from Kitsikoudis et al. (2014b). The SVR implementation supplied this study with additional results for comparison, and the methodology followed is presented analytically in Iliadis et al. (2011). In SVR the aim is to determine a function that has at most ε deviation from the actual measurements, and at the same time is as flat as possible. The optimal parameters for this SVR implementation are shown in Table 2, where C determines the trade-off between the flatness of the generated function and the tolerated deviations, and γ defines the point where the empiric error is related to complexity.

Table 2. Optimal parameters for the SVR implementation

		C	γ	ε			C	γ	ε
	(a)	1240	0.11875	0.125		(a)	2.8	2.8	0.075
Sand-bed rivers	(b)	5	0.9	0.00625	Gravel-bed rivers	(b)	1.6	9.2	0.1125
	(c)	260	0.625	0.1625		(c)	1.7	9.2	0.11875

4 APPLICATIONS AND RESULTS

The employed data originate from reliable compilations, which have provided data for several comparison studies. Brownlie's (1981a) compilation provided the bed-material load data for sand-bed rivers, while an extensive field campaign in mountainous streams in Idaho (King et al., 2004, Kitsikoudis and Hrissanthou, 2013; Kitsikoudis et al., 2014b) provided the data for the bed load study in gravel-bed rivers.

The optimal machine learning model configuration is considered to be the one that generates the better results in the validation set. As a result, the testing set remains unused during the training process and the predictive capability of the considered model is consequently assessed. The generated results P_i compared to the respective observed ones O_i and to the mean observed value $\bar{O}$ are evaluated on the basis of root mean square error ($RMSE$),

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{N}(O_i - P_i)^2}{N}} \tag{9}$$

mean absolute error (MAE),

$$\mathrm{MAE} = \frac{\sum_{i=1}^{N}|O_i - P_i|}{N} \tag{10}$$

Table 3. Performance evaluation of machine learning for bed-material load prediction in sand-bed rivers, in terms of ppm

		$RMSE$ (ppm)	MAE (ppm)	MNE (%)	CE	$DR_{0.5-2}$ (%)	$DR_{0.25-4}$ (%)
Training set	ANN(a)	391.11	212.54	44.05	0.787	85.99	98.46
	ANN(b)	413.32	219.36	48.07	0.762	81.77	96.74
	ANN(c)	418.75	228.82	50.66	0.756	81.57	96.74
	ANFIS(a)	426.94	229.84	49.15	0.746	82.15	97.70
	ANFIS(b)	401.31	222.57	48.79	0.776	82.34	97.50
	ANFIS(c)	413.08	222.60	51.37	0.762	81.19	97.50
	SR(a)	449.56	242.12	52.93	0.718	79.27	96.55
	SR(b)	460.93	246.94	53.90	0.704	78.89	96.55
	SR(c)	435.51	235.52	56.25	0.736	78.12	96.16
	SVR(a)	406.99	221.87	49.46	0.769	81.77	97.50
	SVR(b)	415.49	218.48	46.40	0.759	81.00	97.12
	SVR(c)	421.42	231.72	53.32	0.753	83.88	96.74
Validation set	ANN(a)	430.07	230.99	42.68	0.722	81.61	98.28
	ANN(b)	419.22	222.30	46.03	0.736	81.61	97.70
	ANN(c)	430.02	229.06	44.19	0.722	81.61	98.28
	ANFIS(a)	457.61	236.59	43.97	0.685	81.61	96.55
	ANFIS(b)	455.76	238.01	43.81	0.688	80.46	95.98
	ANFIS(c)	478.84	247.33	47.47	0.655	78.16	94.83
	SR(a)	477.26	251.95	48.20	0.657	77.01	94.25
	SR(b)	477.49	247.09	48.10	0.657	79.31	94.83
	SR(c)	493.99	258.03	50.33	0.633	78.16	94.25
	SVR(a)	468.61	244.40	44.64	0.670	83.91	97.13
	SVR(b)	445.72	225.83	43.08	0.701	79.89	95.40
	SVR(c)	470.21	242.67	44.74	0.668	83.91	96.55
Testing set	ANN(a)	452.04	207.87	45.62	0.755	79.77	95.95
	ANN(b)	478.06	234.57	45.52	0.726	80.35	96.53
	ANN(c)	452.94	219.73	45.37	0.754	80.92	98.27
	ANFIS(a)	492.74	235.89	43.51	0.709	80.92	97.11
	ANFIS(b)	498.98	244.87	45.95	0.701	80.92	97.69
	ANFIS(c)	495.15	240.15	45.81	0.706	78.61	97.11
	SR(a)	515.04	250.63	49.24	0.682	78.03	97.69
	SR(b)	515.63	248.62	47.04	0.681	80.92	97.69
	SR(c)	506.36	248.32	48.58	0.693	77.46	96.53
	SVR(a)	494.78	240.61	47.40	0.706	80.35	96.53
	SVR(b)	504.42	242.25	44.59	0.695	79.19	97.11
	SVR(c)	495.45	239.54	47.73	0.706	76.88	97.11
	Ackers and White (1973)	720.56	359.40	51.02	0.381	57.56	90.12
	Brownlie (1981b)	648.44	322.17	45.78	0.496	68.79	94.22
	Engelund and Hansen (1967)	637.96	336.12	57.43	0.522	67.07	91.62
	Karim and Kennedy (1990)	767.41	383.88	64.13	0.294	62.43	90.75
	Molinas and Wu (2001)	726.47	387.76	68.80	0.367	58.38	91.33
	Yang (1973)	724.24	396.81	61.19	0.371	42.20	72.25

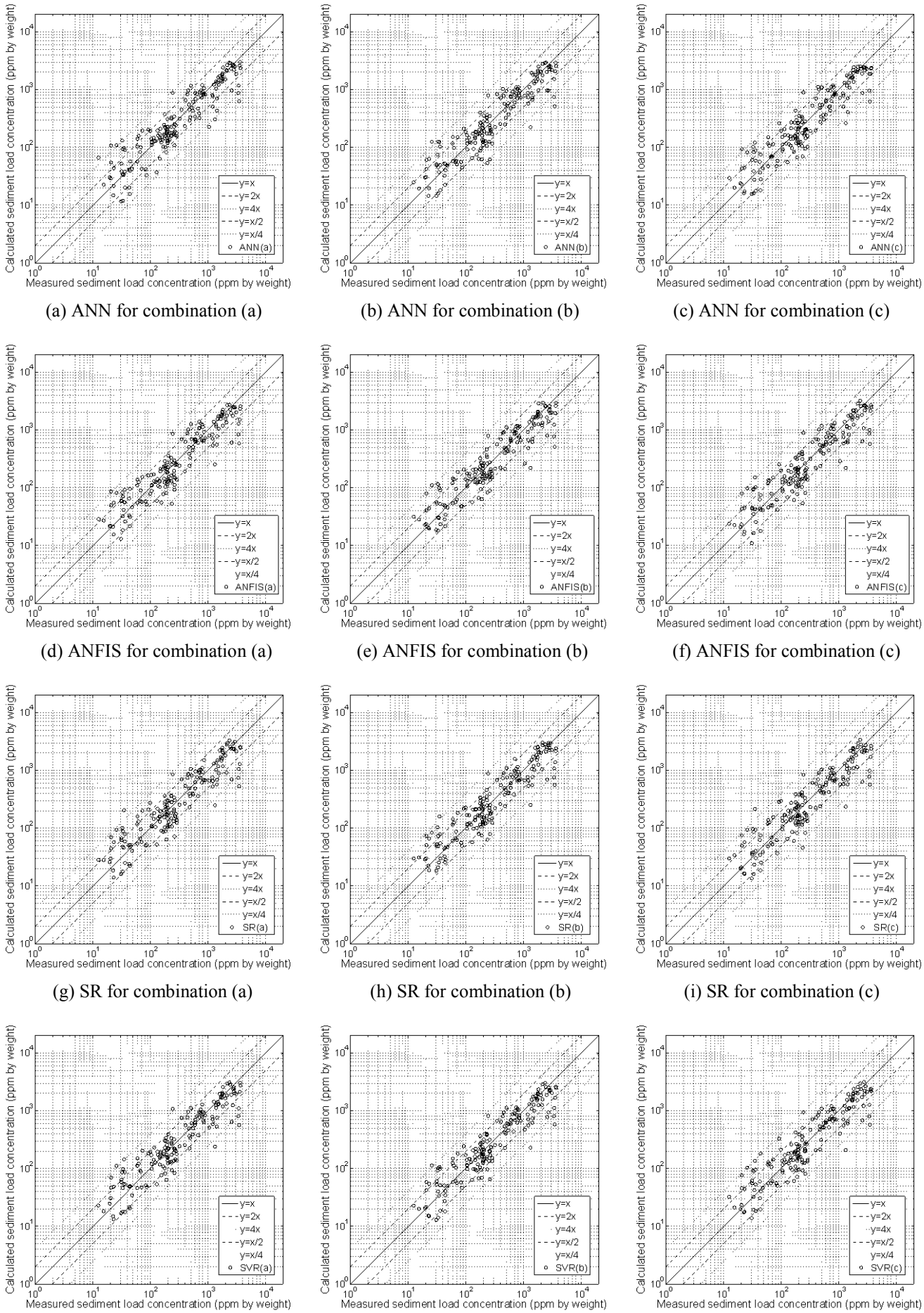
mean normalized error (MNE),

$$MNE = \frac{100}{N}\sum_{i=1}^{N}\left|\frac{O_i - P_i}{O_i}\right| \tag{11}$$

Nash and Sutcliffe (1970) coefficient of efficiency (CE),

$$\mathrm{CE} = 1\text{-}\frac{\sum_{i=1}^{N}(O_i\text{-}P_i)^2}{\sum_{i=1}^{N}(O_i\text{-}\bar{O})^2} \tag{12}$$

and discrepancy ratio (DR), which is the percentage of the calculated sediment transport rates that lie within one half (or a quarter, or a tenth) and two (or four, or ten) times the respective measured sediment transport rates. Additionally, models with simpler structure were preferred from complicated ones with



(a) ANN for combination (a) (b) ANN for combination (b) (c) ANN for combination (c)

(d) ANFIS for combination (a) (e) ANFIS for combination (b) (f) ANFIS for combination (c)

(g) SR for combination (a) (h) SR for combination (b) (i) SR for combination (c)

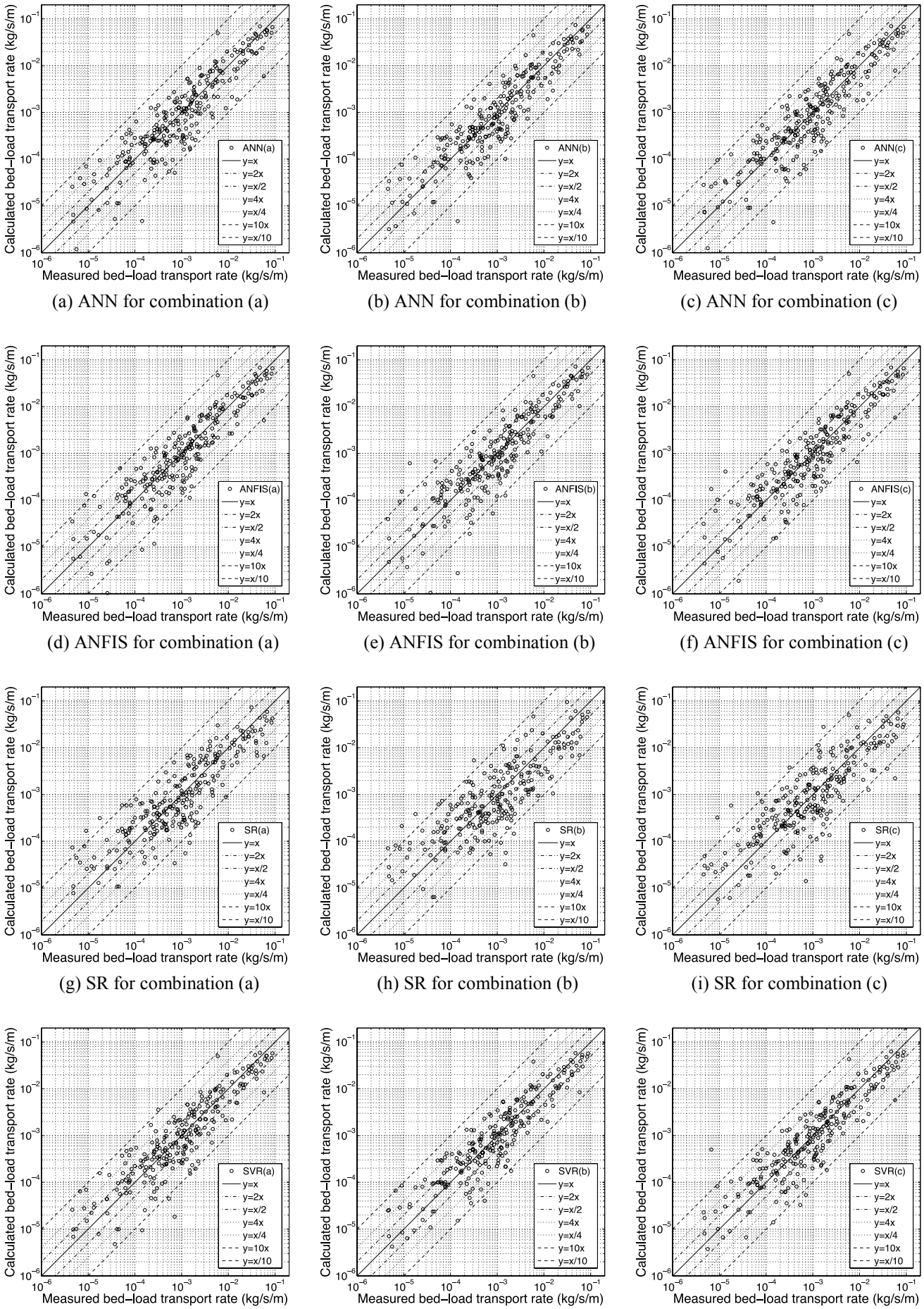
(j) SVR for combination (a) (k) SVR for combination (b) (l) SVR for combination (c)

Figure 1. Scatter plots of the measured versus the respective machine learning calculated bed-material load in sand-bed rivers, for the test set, in ppm.

Table 4. Performance evaluation of machine learning for bed load prediction in gravel-bed rivers, in terms of kg/s/m

		$RMSE$ (kg/s/m)	MAE (kg/s/m)	MNE (%)	CE	$DR_{0.5-2}$ (%)	$DR_{0.25-4}$ (%)	$DR_{0.1-10}$ (%)
Training set	ANN(a)	0.0055	0.0021	80.74	0.840	67.47	92.53	98.53
	ANN(b)	0.0063	0.0023	79.60	0.793	64.00	92.80	98.53
	ANN(c)	0.0060	0.0022	81.61	0.812	67.47	92.00	98.40
	ANFIS(a)	0.0053	0.0020	57.79	0.855	75.60	96.53	99.73
	ANFIS(b)	0.0057	0.0021	60.77	0.833	73.20	95.60	99.73
	ANFIS(c)	0.0056	0.0020	68.64	0.835	70.27	94.13	99.33
	SR(a)	0.0097	0.0033	129.46	0.506	55.33	83.87	96.93
	SR(b)	0.0090	0.0034	173.99	0.578	45.87	77.07	93.73
	SR(c)	0.0094	0.0034	145.76	0.542	50.67	80.80	94.80
	SVR(a)	0.0062	0.0023	97.87	0.799	66.40	89.60	97.33
	SVR(b)	0.0065	0.0023	88.43	0.780	72.67	93.20	98.40
	SVR(c)	0.0062	0.0023	87.45	0.798	72.00	91.87	97.60
Validation set	ANN(a)	0.0066	0.0026	80.77	0.800	60.00	88.40	98.40
	ANN(b)	0.0068	0.0027	80.24	0.788	63.20	88.00	98.40
	ANN(c)	0.0067	0.0026	83.77	0.791	60.00	88.80	98.00
	ANFIS(a)	0.0063	0.0024	84.06	0.816	61.20	86.00	96.00
	ANFIS(b)	0.0064	0.0025	103.74	0.812	60.80	88.00	96.40
	ANFIS(c)	0.0065	0.0025	108.21	0.807	58.40	85.20	96.00
	SR(a)	0.0096	0.0033	113.26	0.569	52.40	80.00	97.20
	SR(b)	0.0089	0.0036	141.32	0.637	43.20	75.60	95.20
	SR(c)	0.0099	0.0037	154.18	0.548	46.40	78.00	94.80
	SVR(a)	0.0067	0.0024	79.34	0.792	67.60	92.40	98.00
	SVR(b)	0.0071	0.0025	71.28	0.768	72.00	92.40	99.20
	SVR(c)	0.0067	0.0025	75.18	0.794	72.40	92.40	99.20
Testing set	ANN(a)	0.0067	0.0027	82.64	0.779	58.00	84.40	99.20
	ANN(b)	0.0068	0.0027	83.56	0.777	58.80	85.20	98.80
	ANN(c)	0.0068	0.0028	86.63	0.774	57.20	83.60	98.00
	ANFIS(a)	0.0068	0.0027	90.16	0.777	57.20	83.20	95.60
	ANFIS(b)	0.0068	0.0027	114.21	0.777	57.60	85.60	95.60
	ANFIS(c)	0.0070	0.0027	104.72	0.765	58.40	84.80	96.80
	SR(a)	0.0092	0.0035	120.50	0.594	50.00	80.80	96.40
	SR(b)	0.0098	0.0040	155.38	0.536	40.40	73.60	94.80
	SR(c)	0.0100	0.0040	149.40	0.511	45.20	77.60	93.60
	SVR(a)	0.0071	0.0028	93.41	0.756	59.60	84.40	97.60
	SVR(b)	0.0071	0.0028	84.79	0.753	58.40	86.00	99.20
	SVR(c)	0.0072	0.0028	111.01	0.747	58.00	85.60	97.60
	Bagnold (1980)	0.0207	0.008	234.9	-1.084	0	0	0.80
	Meyer-Peter and Mueller (1948)	0.0157	0.006	100	-0.195	0	0	0
	Parker (1979)	4.276	1.334	275957	-88655	3.20	5.60	12.80
	Recking (2013)	0.0223	0.0098	965.48	-1.406	27.60	51.60	76.00
	Schoklitsch (1962)	2.402	1.518	570802	-27979	0	0	0

similar or even slightly better performance, according to the principle of parsimony. Tables 3 and 4 show the results of the implementation of the aforementioned techniques in sand-bed and gravel-bed rivers, respectively, for all the input combinations of Table 1, and the results obtained from some well-known sediment transport functions for the test set data, from which can be inferred the superiority of machine learning. Figures 1 and 2 depict the scatter plots between the measured and the respective calculated bed-material load in sand-bed rivers and bed load in gravel-bed rivers, respectively, for the test set data. All three input combinations provide relatively equally good results, depending on the regression scheme utilized, despite the different physical meaning of these variables, namely the unit stream power, stream power, and shear stress. Finally, it can be seen that ANNs perform better than the other techniques in sand-bed rivers, while in gravel-bed rivers they produce similar results to ANFIS and SVR. SR generates the least good results.



(a) ANN for combination (a) (b) ANN for combination (b) (c) ANN for combination (c)

(d) ANFIS for combination (a) (e) ANFIS for combination (b) (f) ANFIS for combination (c)

(g) SR for combination (a) (h) SR for combination (b) (i) SR for combination (c)

(j) SVR for combination (a) (k) SVR for combination (b) (l) SVR for combination (c)

Figure 2. Scatter plots of the measured versus the respective machine learning calculated bed load in gravel-bed rivers, for the test set, in kg/s/m.

5 CONCLUSIONS

This study demonstrated the potential of machine learning in the context of bed-material load and bed load prediction in sand-bed and gravel-bed rivers, respectively, based on several independent variables. Four techniques have been utilized, with ANNs providing the better results, followed closely by ANFIS and SVR, while SR provided the least good results. All these models performed significantly better than some of the commonly used sediment transport formulae. Three different input combinations were used, based mainly on unit stream power, stream power, and shear stress, and all of them provided similarly good results, thus highlighting the importance of the regression scheme. Finally, the data-driven models were able to predict sediment transport rates without the need to set a criterion for the initiation of sediment movement, hence avoiding erroneous zero transport predictions, a common problem of the bed load quantification in gravel-bed rivers.

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