

# TEMPORAL VARIATION OF SCOUR DEPTH AT NONUNIFORM CYLINDRICAL PIER WITH UNEXPOSED FOUNDATION

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The temporal development of the nonuniform pier scour is quite different from that for the uniform pier due to the effect of foundation. Based on the primary vortex concept and the volumetric sediment transport theory, a methodology is proposed to characterize the rate of enlargement in scour hole and the entire scouring process at nonuniform piers with a discontinuous surface located below the initial bed level. The scouring processes includes three Zones, namely Zone 1 where the scouring phenomenon is similar to that for a uniform pier without the influence of foundation, Zone 2 where scour depth remains unchanged, and Zone 3 where the pier geometry affects the scouring process. A concept of superposition using an effective pier diameter is proposed to simulate the scouring process for Zone 3. In general, the simulated results based on the proposed method correspond well with the experimental data collected by different investigators.

**Key Words :** *nonuniform piers; clear-water scour; sediment transport; horseshoe vortex; foundation*

## 1. INTRODUCTION

Most bridge scour researchers investigated the scour process at a uniform pier. However, in reality a lot of bridge piers are nonuniform in shape. The temporal variation of scour depth at nonuniform piers is an important research topic. For example, the time factor may play a significant role in the determination of the closure time for cross-river bridges.

Detailed experimental investigations on the nonuniform pier scouring process had previously been conducted by Melville and Raudkivi<sup>1)</sup>, and Oliveto et al.<sup>2)</sup> A numerical model was developed by Imamoto and Ohtoshi<sup>3)</sup> to simulate the temporal variation of the local scour at a nonuniform cylindrical pier with a protruding foundation. However, Imamoto and Ohtoshi's model had not

been tested with the experimental data. Studies related to the modeling of the temporal variations of scour depths at nonuniform piers with unexposed foundations are still limited for the present.

The main objective of this study is to develop a semi-empirical model based on the concept of the primary vortex and the sediment transport theory to estimate the temporal variations of scour depth at nonuniform piers with unexposed foundations.

## 2. EXPERIMENTS

The laboratory experiments were conducted in a 17-m-long, 0.6-m-wide, and 0.6-m-deep re-circulating flume. The median size of the sediment was 0.52 mm. The test reach was 7.5 m long and 0.25 m deep. The test structure was positioned near the center of the reach,

approximately 10 m downstream of the flume entrance. A flow straightener at the upstream end of the flume helped producing a nearly uniform flow at the entrance. The pier models were made of plexiglass. Scour depth was measured with a periscope inserted in the pier, while the water surface elevation was determined by a point gauge.

The average approaching flow velocity was measured by an Acoustic Doppler Velocimeter (ADV). The vertical velocity profile was measured at the centerline of the flume 2 m upstream of the pier. To release air in the sediment, the flume was slowly filled with water and left undisturbed for approximately 24 hours before the experiment. According to Melville and Chiew<sup>4)</sup>, the equilibrium time  $t_e$  can be defined as the time at which the scour hole develops to equilibrium scour depth  $d_{se}$  at which the increase of scouring rate does not exceed 5% of the pier diameter in the succeeding 24 hours period.

The clear-water scour condition was maintained for all the experiments. The experimental conditions and the measured results are summarized in Table 1. For comparison, the laboratory data collected by Melville and Raudkivi<sup>1)</sup> are also included in the table. The median particle size in the experiments of Melville and Raudkivi<sup>1)</sup> was 0.80 mm.

**Table 1** Summary of test conditions and measured scour depths

Series (—)	$D$ (mm)	$D^*$ (mm)	$Z$ (mm)	$h$ (mm)	$\frac{U}{U_c}$ (—)	$d_{se}$ (mm)	$t_e$ (min)
S1	35	50	25	190	0.9	55	6644
S5	35	50	50	175	0.9	78	4554
S10	20	50	10	175	0.9	26	2034
S11	35	50	25	170	0.8	34	4915
MR-D1	30	81	35	200	1.0	38	1153
MR-D2	30	81	30	200	1.0	42	1663
MR-D3	30	81	25	200	1.0	65	2845

$D$  = pier diameter;  $D^*$  = foundation diameter;  $Z$  = distance between the initial bed and the top of foundation;  $h$ : approach flow velocity;  $U$  = approaching flow velocity;  $U_c$  = critical velocity;  $d_{se}$  = equilibrium scour depth;  $t_e$  = equilibrium scour time

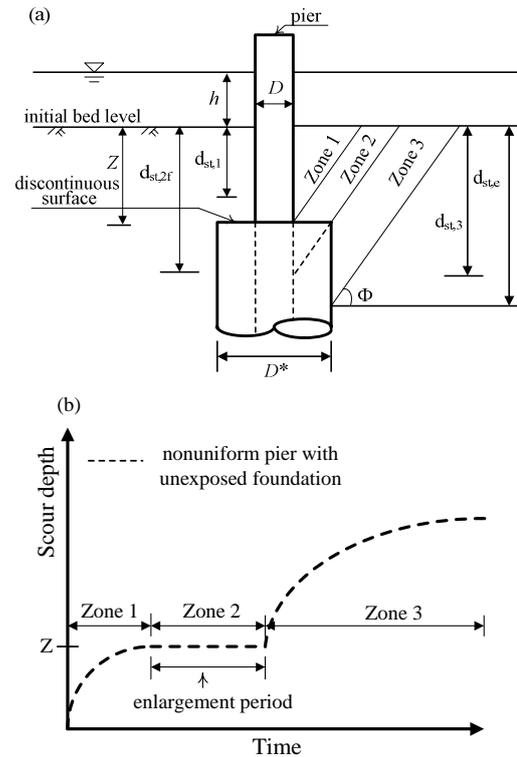
Note: S—current study, MR—Melville and Raudkivi<sup>1)</sup>.

### 3. SIMULATION MODEL

#### (1) Scouring process

The proposed model for simulating the scour process at a nonuniform pier with an unexposed foundation consists of three components as shown in Fig. 1. First, Zone 1 corresponds to the scouring process for a uniform pier without the influence of the foundation. Second, Zone 2 represents a period that the scour depth remains unchanged, and is equal to the distance ( $Z$ ) between the initial river bed and

the top of foundation. Finally, the scour depth increases again in Zone 3, where the pier geometry affects the scouring process.



**Fig.1** Schematic diagram of scouring process at nonuniform pier with unexposed foundation.

The scouring may reach equilibrium in Zone 1, Zone 2, or Zone 3 depending on the approach flow intensity ( $U/U_c$ ), bed material ( $d_{50}$ ), distance ( $Z$ ) between the top of foundation and the initial bed, and the pier geometry ( $D/D^*$ ). Due to the space limit, only the cases that scouring reached equilibrium in Zone 1 and Zone 3 are discussed in this paper.

On the basis of the primary vortex concept and the volumetric sediment transport theory, Mia and Nago<sup>5)</sup> proposed a semi-empirical model to compute the time variations of scour at uniform piers. In this study, the model is modified to describe the scour rate ahead of the pier nose for a nonuniform pier under the steady clear-water scour conditions.

#### (2) Primary vortex and shear velocity variations

Kothyari et al.<sup>6)</sup> assumed that the horseshoe vortex is the primary scouring mechanism and its diameter can be estimated by  $D_V = 0.28h(D/h)^{0.85}$ , where  $h$  is the approach flow depth, and  $D$  is the pier diameter. The shape of the scour hole was approximated by an inverted cone during the process of scouring. The cross-section area of the primary vortex at any time  $t$  can be expressed as  $A_t = 0.25\pi D_V^2 + 0.5d_{st}^2 / \tan \Phi$ ,

where  $d_{st}$  is the scour depth at time  $t$ , and  $\Phi$  is the angle of scour hole. The angle of scour hole was assumed to be the angle of repose of sediment.

As the scour progresses, the bed-shear velocity at the pier nose decreases with an increase in the cross-sectional area of the primary vortex, and it can be estimated as:

$$u_{*t} = 3.3u_{*s} \left( 0.25\pi D_V^2 / A_t \right)^C \quad (1)$$

where  $u_{*t}$  = shear velocity at pier nose at time  $t$ ;  $u_{*s}$  = shear velocity of the approach flow, and  $C$  was estimated to be 0.29 in Zone 1 and Zone 2 by the experimental data.

### (3) Volumetric sediment transport rate at piers

According to Mia and Nago<sup>5)</sup>, the sediment transport rate at a pier can be described by using the concepts of Yalin's<sup>7)</sup> bed-load transport function and the volume of the scour hole developed with time around the pier. The dimensionless sediment transport rate can be expressed as  $\Pi q_{st} = q_{st} / d_{50} u_{*t} = k s_t [1 - (1/as_t) \ln(1 + as_t)]$ , where  $q_{st}$  = the volumetric sediment transport rate per unit movable width of the scour hole;  $\Pi q_{st}$  = the dimensionless value of  $q_{st}$ ;  $d_{50}$  = median grain size;  $k$  = constant to be determined by experiments;  $s_t = (u_{*t}/u_{*c})^2 - 1$ ;  $u_{*c}$  = critical shear velocity;  $a = 2.45 \sqrt{\theta_c} / G_s^{0.4}$ ;  $\theta_c = u_{*c}^2 / [(G_s - 1)gd_{50}]$ ;  $G_s = \rho_s / \rho$ ;  $g$  = gravity acceleration;  $\rho_s$  and  $\rho$  are the densities of sediment and water, respectively.

Mia and Nago<sup>5)</sup> suggested a  $k$  value of 1.8. However, it has to be mentioned that the exponent in the expression of  $s_t$  was 1.0 in Mia and Nago's paper, which is supposed to be 2.0. Accordingly, the  $k$  value should also be corrected to 1.23, i.e.  $q_{st} = 1.23 s_t [1 - (1/as_t) \ln(1 + as_t)]$ .

For any number of time step  $n$ , the volume of the scour hole per unit moveable width of the scour hole  $q_{vt}$  can be expressed as  $q_{vt} = \sum_{i=0}^n q_{st_i}$ , where  $i$  = time step index. Assuming the shape of the scour hole remains almost unchanged and the scour hole is approximately an inverted cone during the scouring process, the scour depth at time  $t$  can be expressed as

$$d_{st} = \sqrt{\frac{6 \tan \Phi}{\pi} q_{vt} + \frac{9D^2 \tan^2 \Phi}{16} - \frac{3D \tan \Phi}{4}} \quad (2)$$

As shown in Fig. 1, for Zone 1 the scouring process is similar to that under a uniform pier

condition. When the scour depth reaches  $Z$  (distance between the initial bed and the top of foundation), i.e.  $d_{st,1} = Z$ , the scouring process enters Zone 2.

For Zone 2, the horseshoe vortex enlarges the scour hole but the scour depth remains unchanged. To reflect the retardation effect of the discontinuous surface, the  $k$  value in Yalin's<sup>7)</sup> equation was modified based on the experimental data collected in this study and those by Melville and Raudkivi<sup>1)</sup> as follows:

$$k_{zone2} = \exp[-58.2788(\frac{Z}{D}) - 0.0004(\frac{D^*}{d_{50}})^2 + 0.6037(\frac{Z}{D})(\frac{D^*}{d_{50}})] \quad (3)$$

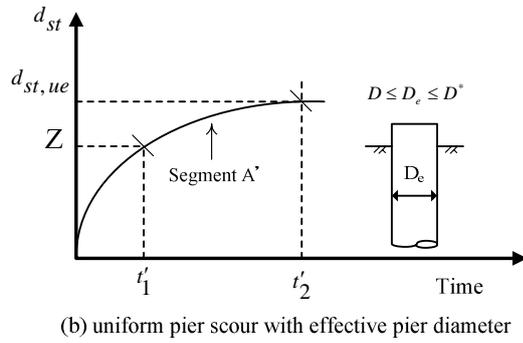
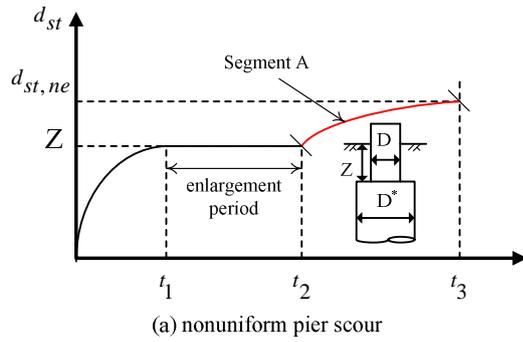
where all the variables are as defined previously (see Fig. 1). Eq. (3) indicates that  $k_{zone2}$  decreases with an increase of parameter  $Z/D$  and  $D^*/d_{50}$ , respectively. Since the volumetric sediment transport rate  $q_{st}$  is proportional to  $k$  value, physically the time to reach Zone 3 from Zone 2 decreases with an increase of  $k_{zone2}$  value.

When the scour depth  $d_{st,2f}$  reaches a value of  $Z + 0.5(D^* - D) \tan \phi$ , the scouring process enters Zone 3. It should be noted that the  $d_{st,2f}$  only corresponds to a fiction scour depth. For Zone 3, the horseshoe vortex was affected by the discontinuous surface, a scheme is herein proposed for computing the scour-depth evolution. As shown in Fig. 2(a), the scour depth evolves from  $Z$  to  $d_{st,ne}$  along segment A for Zone 3 with a nonuniform pier.

Melville and Raudkivi<sup>1)</sup> proposed a concept of effective pier diameter ( $D_e$ ) to estimate the scour depth for nonuniform cylindrical pier. This concept is adopted to compute the scour depth in this study. To determine the equilibrium scour depth, segment A in Fig. 2(a) is assumed to be equivalent to segment A' in Fig. 2(b) for a uniform pier with an effective pier diameter  $D_e$ . However, Melville and Raudkivi<sup>1)</sup> showed that use of  $D_e$  led to conservative estimate of scour depth for nonuniform cylindrical piers. Therefore, exponent  $C$  in Eq. (1) should be modified to reflect the scouring process in Zone 3. From the experimental data of Melville and Raudkivi<sup>1)</sup> and current study, the regression equation for exponent  $C$  in Zone 3 can be modified to be:

$$C_{zone3} = \exp[-3.15(\frac{U}{U_c}) - 23.19(\frac{Z}{D}) + 0.037(\frac{D^*}{d_{50}}) + 2.42(\frac{Z}{D})^2 + 0.19(\frac{Z}{D})(\frac{D^*}{d_{50}})] \quad (4)$$

Physically the time to equilibrium decreases with an increase of  $C$  value for Zone 3.



**Fig. 2** Schematic diagram for computing scour-depth evolution in Zone 3.

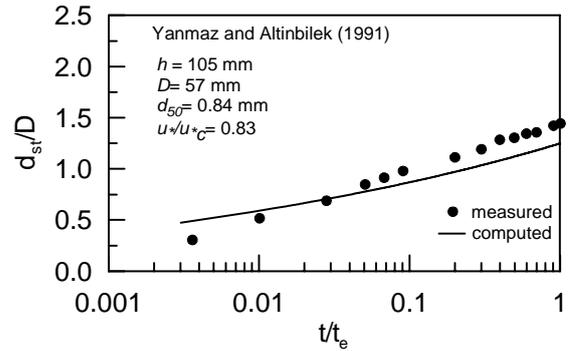
The computation of temporal variation of scour depth at nonuniform cylindrical pier with an unexposed foundation is given in a flowchart in Appendix A.

#### 4. RESULTS

The coefficients of  $k$  and  $C$  for Zone 1~Zone 3 adopted in the numerical simulation are summarized in Table 2. Fig. 3 shows a comparison of the results from the model and the experimental data of Yanmaz and Altinbilek<sup>8)</sup> for a uniform pier under the steady clear-water scour condition. The simulated temporal variation of scour depth by the present model is satisfactory. The results also reveal that the new calibration value  $k=1.23$  is proper for simulating the time development of the pier scour with uniform pier such as Zone 1 condition.

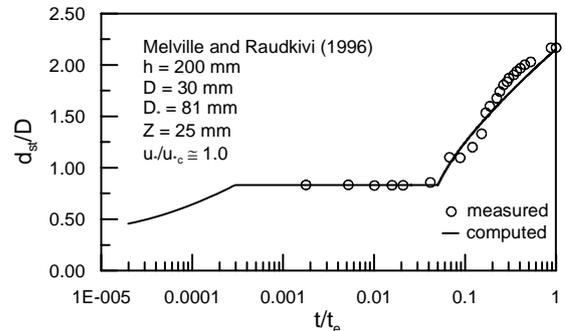
**Table 2** Coefficients of  $k$  and  $C$

coefficient \ region	region		
	Zone 1	Zone 2	Zone 3
$k$	1.23	Eq. (3)	1.23
$C$	0.29	0.29	Eq. (4)

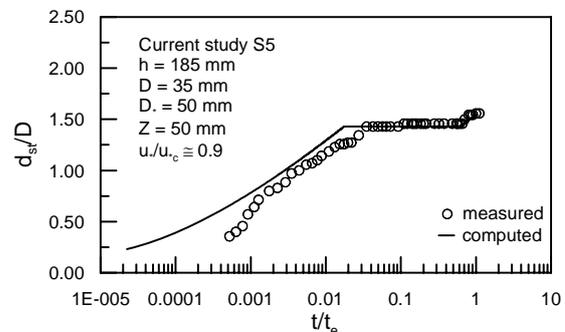


**Fig. 3** Comparison of the results from the present model and the experimental data of Yanmaz and Altinbilek (1991) for a uniform pier.

Furthermore, the computed scour depths with the present model are also compared with the scour depths observed by different investigators. Figs. 4 and 5 show some of the calibrated results for Melville and Raudkivi<sup>1)</sup>, and the writer's data. As shown in Fig. 4, the model gives very good prediction for the experimental data collected by Melville and Raudkivi<sup>1)</sup>. Fig. 5 indicates that there are certain discrepancies between the experimental data and the simulated results particularly during the initial scouring stage. However, the model describes the scouring processes in Zone 2 and Zone 3 reasonably well.



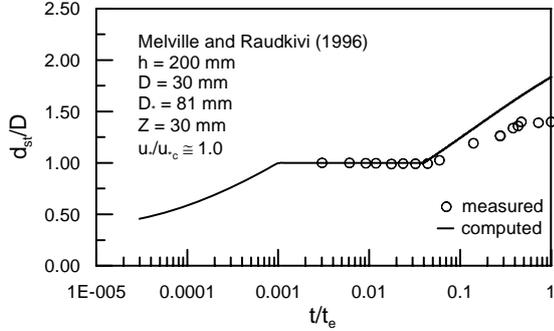
**Fig. 4** Temporal variation of scour depth for nonuniform pier (calibration, MR-D3).



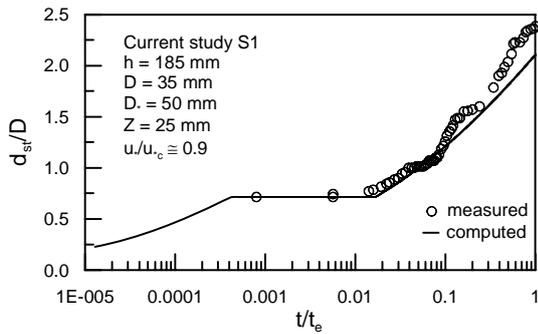
**Fig. 5** Temporal variation of scour depth for non-uniform pier (calibration, S5).

Figs. 6 and 7 give the additional simulated results for two cases which were not included in the development of the empirical formulas for  $k$  and  $C$ ,

i.e., Eqs. (3) and (4). As the experimental data available for the estimation of coefficients  $k$  and  $C_{Zone3}$  are still limited, some discrepancies between the measured and predicted values can be detected. Nevertheless, in general, the model predicts the time evolutions of the scour depth reasonably well for both uniform and nonuniform piers with unprotruding foundations.



**Fig. 6** Temporal variation of scour depth for non-uniform pier (prediction, MR-D2).

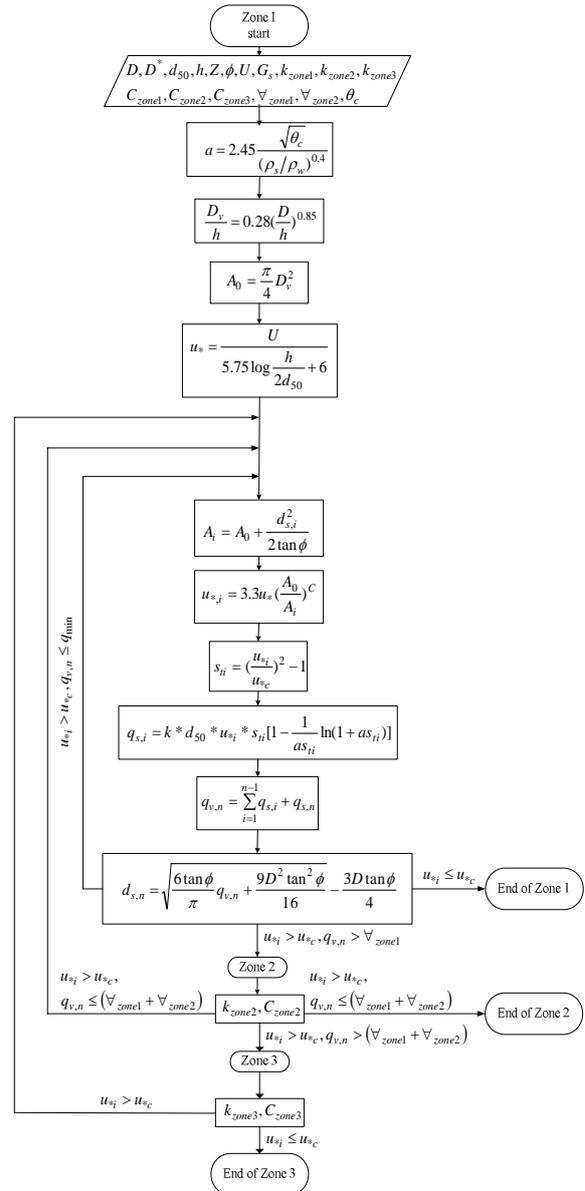


**Fig. 7** Temporal variation of scour depth for non-uniform pier (prediction, S1).

## 5. CONCLUSIONS

The time variations of scour depth for both uniform and nonuniform piers under clear-water scour conditions have been modeled. The scouring process at the pier nose has been simulated using the concepts of primary vortex and the volumetric sediment transport theory. Empirical formulas for the estimation of the coefficient of sediment transport rate ( $k$ ) and the scouring rate coefficient  $C$  in Zone 1~Zone 3 were developed using the available laboratory data. A concept of superposition using an effective pier diameter is proposed to simulate the scouring process for Zone 3. In addition, based on the experimental data collected in this study and those reported in the literature, the simulation results indicate that the proposed model predicts the temporal variations of scour depths reasonably well for both uniform and nonuniform piers with unexposed foundations.

## Appendix A Flowchart for scour depth computation



Note:

$v_{zone1}$  = volume of Zone 1

$v_{zone2}$  = volume of Zone 2

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