

INSTABILITY OF STREAM GEOMETRY

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Braided streams are deformed intricately with time and space and broad band scaled streams are coexist in a river reach. Of course, coexist of broad band scaled streams in a river reach causes the complex spatiotemporal change of streams. On the otherhand, bed instability in a stream causes a stream deformation, too. It was discussed here that bed instability at which area affects stream deformation strongly according to the results of numerical analysis. Furthermore, stability characteristics of streams was investigated using numerical analysis and the method how to use the information of predicted braided streams was suggested for river regulation works.

Key Words : Numerical analysis, Instability, bed disturbance, river regulation works

1. INTRODUCTION

Braided streams can be found in rivers where sand bars are formed. Braided streams produce various types of bed geometry (Murray and Paola, 1994) that are inferred to produce diverse physical environments of plants and animals inhabiting rivers (Takebayashi, Egashira & Okabe, 2001). On the other hand, streams tend to form deep scours at certain locations near confining banks. Therefore, it is very important to predict such river features when planning safe river regulation works as well as that of natural riverine ecosystem restoration. In this study, effect of bed small disturbances on spatiotemporal change characteristics of stream geometry is discussed using horizontal two dimensional bed deformation analysis. Furthermore, a use of bed deformation analysis results of braided streams for river regulation works is suggested.

2. ANALYSIS METHOD

Computation of surface flow is performed by the use of the governing equation for the horizontal two-dimensional flow averaged with depth. The conservation of mass, i.e., inflow and outflow of mass by seepage flow, is taken into consideration as

shown in the following equation (Takebayashi & Egashira, 2003).

$$\Lambda \frac{\partial z}{\partial t} + \frac{\partial}{\partial x}(uh) + \frac{\partial}{\partial y}(vh) + \frac{\partial}{\partial x}(u_g h_g) + \frac{\partial}{\partial y}(v_g h_g) = 0 \quad (1)$$

where, t is the time and x and y are the coordinates along the longitudinal and the transverse directions, respectively. Surface flow depth is represented as h , and the seepage flow depth is h_g . u and v represent depth-averaged flow velocity along the longitudinal and the transverse directions, respectively. Depth-averaged seepage flow velocities along the longitudinal and the transverse directions are shown as u_g and v_g , respectively. z is the water surface level. Λ is a porosity parameter related to the relative elevation of bed surface and water surface, wherein $\Lambda = 1$ as $z > z_b$, and $\Lambda = \lambda$ as $z < z_b$, where z_b is the bed level and λ is the porosity of the bed material. Seepage flow is assumed to be a horizontal two dimensional saturated flow. Momentum equations of the surface water are as follows:

$$\frac{\partial}{\partial t}(hu) + \frac{\partial}{\partial x}(huu) + \frac{\partial}{\partial y}(huv) = -gh \frac{\partial}{\partial x}(h + z_b) - \frac{\tau_x}{\rho} + \frac{\partial}{\partial x}(h\sigma_{xx}) + \frac{\partial}{\partial y}(h\tau_{yx}) \quad (2)$$

$$\frac{\partial}{\partial t}(hv) + \frac{\partial}{\partial x}(huv) + \frac{\partial}{\partial y}(hvv)$$

Table 1 Initial bed geometry of the numerical analysis.

Case No.	Calculation time (hr.)	Initial bed geometry
1	9	Flat with random disturbances of a scale about 1/100 of the grain diameter at all over the calculation area.
2	2	Bed geometry of Case1 at 7 hr. with random disturbances of a scale about 1/100 of the grain diameter at all over the calculation area.
3	2	Bed geometry of Case1 at 7 hr. with different spatial distribution of bed disturbances in Case 2, but statistic characteristics of the disturbances in Cases 2 and 3 are identical.
4	2	Bed geometry of Case1 at 7 hr. with a bed disturbance of a scale about 1/100 of the grain diameter is set on a step.
5	2	Bed geometry of Case1 at 7 hr. with a bed disturbance of a scale about 1/100 of the grain diameter is set on a pool.
6	2	Bed geometry of Case1 at 7 hr. with a bed disturbance set at the same location as Case 5, but the height of the disturbance is the half of water depth.

$$= -gh \frac{\partial}{\partial y} (h + z_b) - \frac{\tau_y}{\rho} + \frac{\partial}{\partial x} (h\tau_{xy}) + \frac{\partial}{\partial y} (h\sigma_{yy}) \quad (3)$$

where g is the gravity and ρ is the water density. The conservation of momentum, i.e., inflow and outflow of momentum by seepage flow, is not considered because their values are very small. τ_x and τ_y are the shear stresses along the longitudinal and the transverse directions, respectively. u_b and v_b represent velocity near the bed surface along the longitudinal and the transverse directions, respectively. Velocities near the bed are evaluated using the curvature radius of the streamlines as follows:

$$u_b = u_{bs} \cos \alpha_s - v_{bs} \sin \alpha_s \quad (4)$$

$$v_b = u_{bs} \sin \alpha_s + v_{bs} \cos \alpha_s \quad (5)$$

$$u_{bs} = 8.5u_* \quad (6)$$

$$v_{bs} = -N_* \frac{h}{r} u_{bs} \quad (7)$$

where, N_* is 7.0 (Engelund, 1974) and r is the curvature radius of the stream lines obtained by the depth integrated velocity field as follows (Shimizu & Itakura, 1991):

$$\frac{1}{r} = \frac{1}{(u^2 + v^2)^{3/2}} \left\{ u \left(u \frac{\partial v}{\partial x} - v \frac{\partial u}{\partial x} \right) + v \left(u \frac{\partial v}{\partial y} - v \frac{\partial u}{\partial y} \right) \right\} \quad (8)$$

σ_{xx} , σ_{yy} , τ_{xy} and τ_{yx} are turbulence stresses. Momentum equations of seepage flow are as follows:

$$u_g = -k_{gx} \frac{\partial z}{\partial x} \quad (9)$$

$$v_g = -k_{gy} \frac{\partial z}{\partial y} \quad (10)$$

where k_{gx} and k_{gy} are the coefficients of permeability along the longitudinal and the transverse directions respectively. When the water depth of surface flow becomes less than the mean diameter of the bed material, the surface flow is computed only in consideration of the pressure term and bed shear stress term in the momentum equation of surface

flow (Nagata, 1999).

The temporal change in bed elevation is evaluated by use of the following equation.

$$\frac{\partial z_b}{\partial t} + \frac{1}{1-\lambda} \left(\frac{\partial q_{bx}}{\partial x} + \frac{\partial q_{by}}{\partial y} \right) = 0 \quad (11)$$

q_{bx} and q_{by} are sediment transport rates along the longitudinal and the transverse directions, respectively, as follows (Ashida & Michiue, 1972, Liu, 1991, Kovacs & Parker, 1994):

$$q_{bx} = q_b \cos \beta \quad (12)$$

$$q_{by} = q_b \sin \beta \quad (13)$$

$$q_b = 17 \frac{\rho_s u_*^3}{(\rho_s - \rho) g} \left(1 - \sqrt{K_c} \frac{u_{*c}}{u_*} \right) \left(1 - K_c \frac{u_{*c}^2}{u_*^2} \right) \quad (14)$$

Here ρ_s is the sediment density, u_* is the friction velocity and u_{*c} is the effective shear velocity as follows:

$$u_{*c}^2 = \frac{u^2 + v^2}{\left(6 + 2.5 \ln \frac{h}{d_m (1 + 2\tau_*)} \right)^2} \quad (15)$$

where d_m is the mean diameter of the bed material and u_{*c} is the non-dimensional critical friction velocity. u_{*c} is evaluated by using the Iwagaki's formula (Iwagaki, 1956) which is formulated for uniform bed material.

K_c is a correction factor due to the influence of bed inclination on sediment motion:

$$K_c = 1 + \frac{\left[\left(\frac{\rho}{\rho_s - \rho} + 1 \right) \cos \alpha \tan \theta_x + \sin \alpha \tan \theta_y \right]}{\mu_s} \quad (16)$$

$$\theta_x = \arctan (\partial z_b / \partial x) \quad (17)$$

$$\theta_y = \arctan (\partial z_b / \partial y) \quad (18)$$

The angle of deviation of the near bed flow from the longitudinal direction is as follows:

$$\alpha = \arctan (v_b / u_b) \quad (19)$$

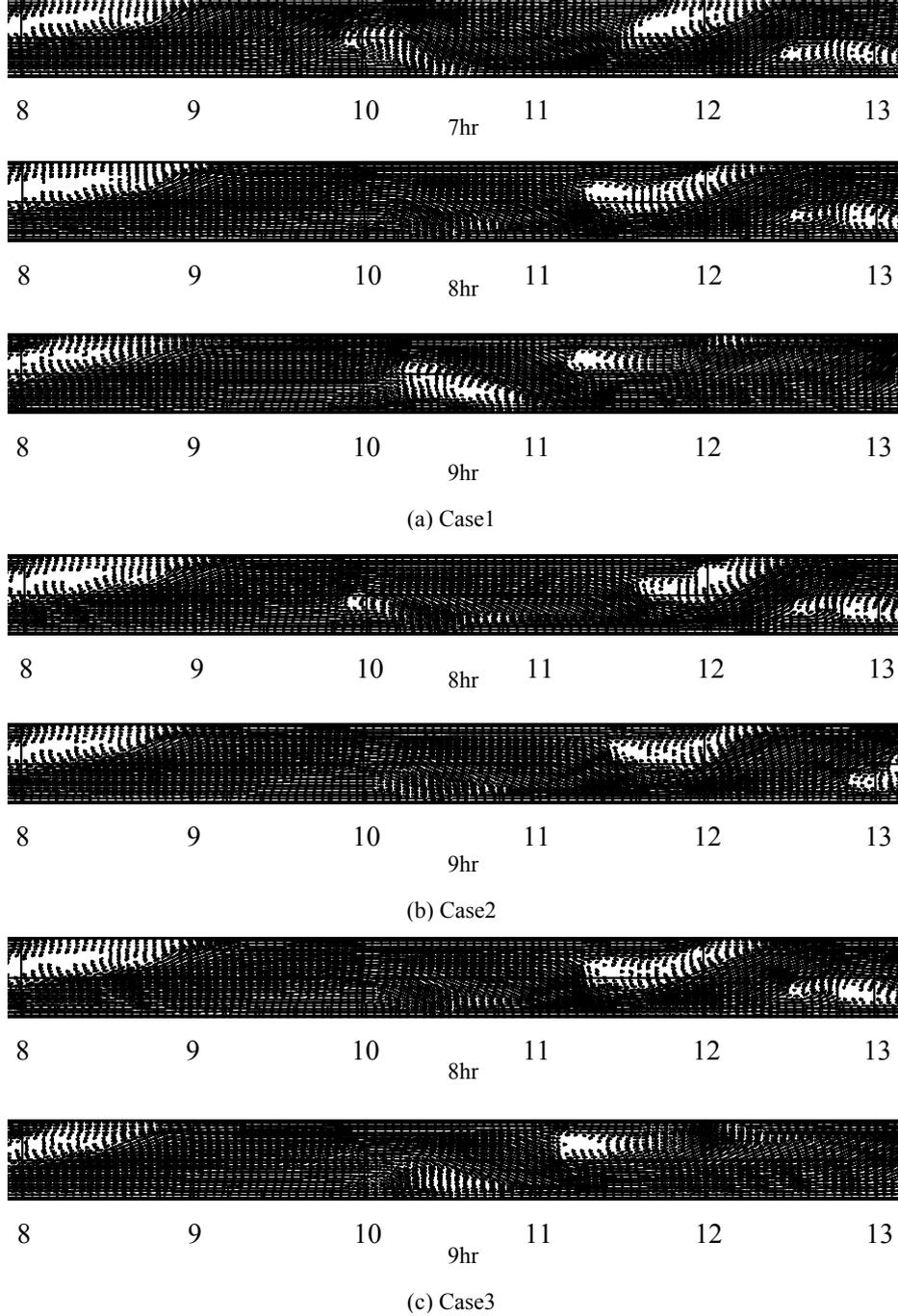


Fig.1 Plane distribution of depth integrated water velocity vector of Case1~Case3 at 7hr, 8hr and 9hr. Water flows from left to right. Velocity vectors are not indicated on islands in figures. For that reason, white and black areas respectively show islands and streams.

The coefficient of static friction is μ_s . The deviation angle of the bed load to the depth averaged flow direction (β), which depends on the flow near the bed and the inclination of the bed, is calculated by the following relations

$$\tan \beta = \frac{\sin \alpha - \Pi \Theta_y \left(\frac{u_{*c}^2}{u_*^2} \right) \tan \theta_y}{\cos \alpha - \Pi \Theta_x \left(\frac{u_{*c}^2}{u_*^2} \right) \tan \theta_x} \quad (20)$$

$$\Pi = K_{ld} + 1/\mu_s \quad (21)$$

$$\Theta_x = \Theta_y + \frac{\rho}{\rho_s - \rho} \cos^2 \theta_x \quad (22)$$

$$\Theta_y = \frac{1}{1 + \tan^2 \theta_x + \tan^2 \theta_y} \quad (23)$$

where K_{ld} ($=0.85$) is the ratio of the lift force to the drag force. The local bed slope is adjusted to be smaller than the angle of repose (Michiue et al 1995).

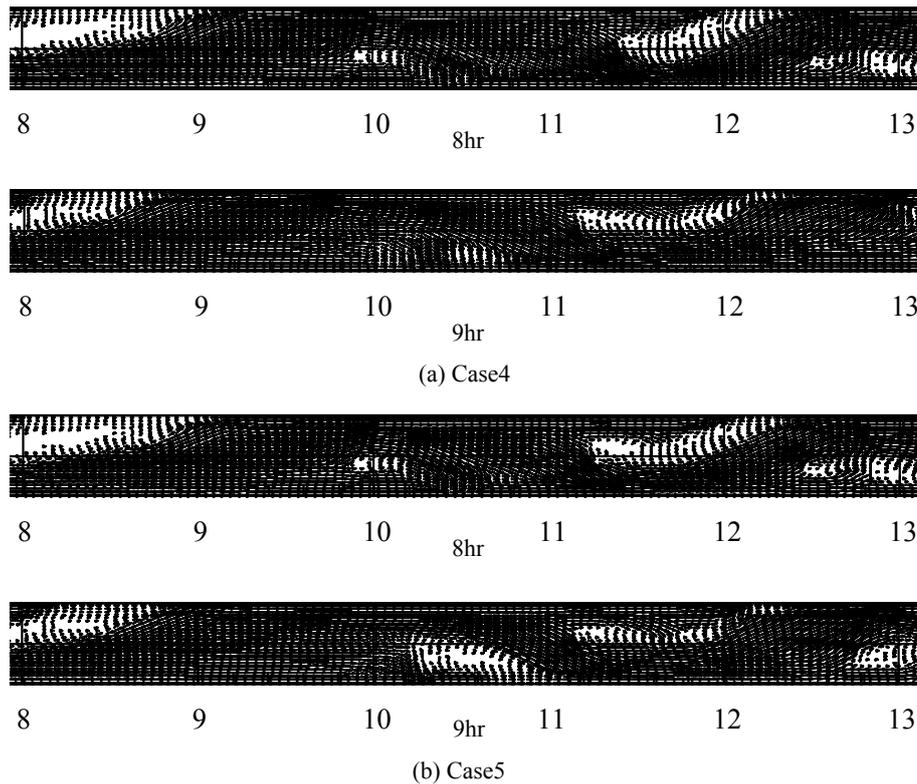


Fig.2 Plane distribution of depth integrated water velocity vector of Case4 and Case5 at 8hr and 9hr. Water flows from left to right. Velocity vectors are not indicated on islands in figures. For that reason, white and black areas respectively show islands and streams.

3. HYDRAULIC CONDITIONS

A straight rectangular channel with 15 m long, 0.4 m wide and rigid side walls is assumed for numerical analyses. The bed slope is 1/100. The initial bed geometry is a flat. A constant amount of water, 0.75 l/s, is supplied from the upstream boundary. As for the sediment discharge at the upstream boundary, those calculated by Ashida et al. (1991) are used based on the hydraulic quantities at the boundary. The mean diameter of bed material is 1.1 mm. The bed material is treated as non-cohesive uniform sediment in the numerical analysis. The width/depth ratio is 62, dimensionless shear stress with respect to the mean grain diameter is 0.041. This hydraulic conditions is located in the formative region of braided streams (Takebayashi, Egashira and Okabe, 2001).

Analysis is performed under 6 conditions. **Table 1** shows the difference of initial bed geometry among 6 conditions. Bed deformation is calculated for 9 hours in Case 1. Flow and bed geometry at 7 hour in Case 1 is used as initial flow and bed geometry in Cases 2-6. Numerical analyses in Cases 2-6 are performed for two hours with different bed disturbances. Random disturbances of a scale about 1/100 of the grain diameter are set to the entire bed at 7 hour in Case 1

in Case 2. Here, the spatial averaged height of the random disturbances is zero. Spatial distribution of bed disturbances in Case 3 is difference from that in Case 2, but statistic characteristics of these two disturbances are identical. In Case 4, a bed disturbance of a scale about 1/100 of the grain diameter is set on a step which has the bed elevation is high and is located at 9.65m from upstream end and 2.4cm from the right bank. In Case 5, a bed disturbance of a scale about 1/100 of the grain diameter is set on a pool which has the bed elevation is low and is located at 9.65m from upstream end and 2.4cm from the left bank. In Case 6, a bed disturbance is set at the same location as Case 5, but the height of the disturbance is the half of water depth.

4. RESULTS AND DISCUSSION

Fig.1 shows horizontal distribution of depth averaged water velocity vectors at 7, 8 and 9 hours in Cases 1-3. The vector plots are reproduced as general illustrations of the flow patterns, and do not need to be examined in detail here. Velocity vectors are not indicated on islands in figures. For that reason, white and black areas respectively show islands and streams. Here, the distributions at 7 hour in Cases 2

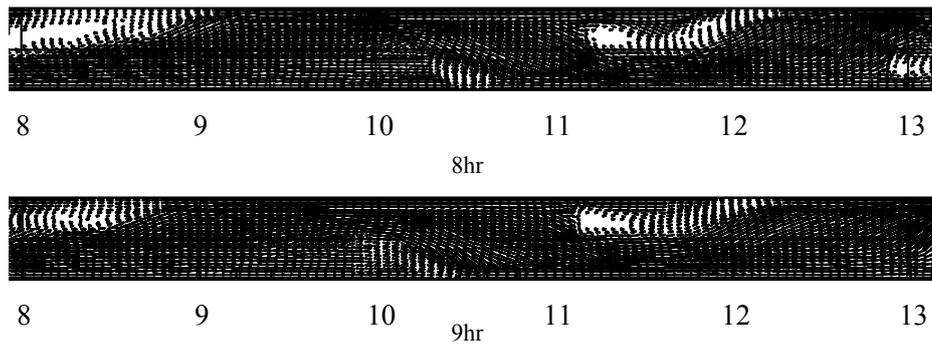


Fig.3 Plane distribution of depth integrated water velocity vector of Case6 at 8hr and 9hr. Water flows from left to right. Velocity vectors are not indicated on islands in figures. For that reason, white and black areas respectively show islands and streams.

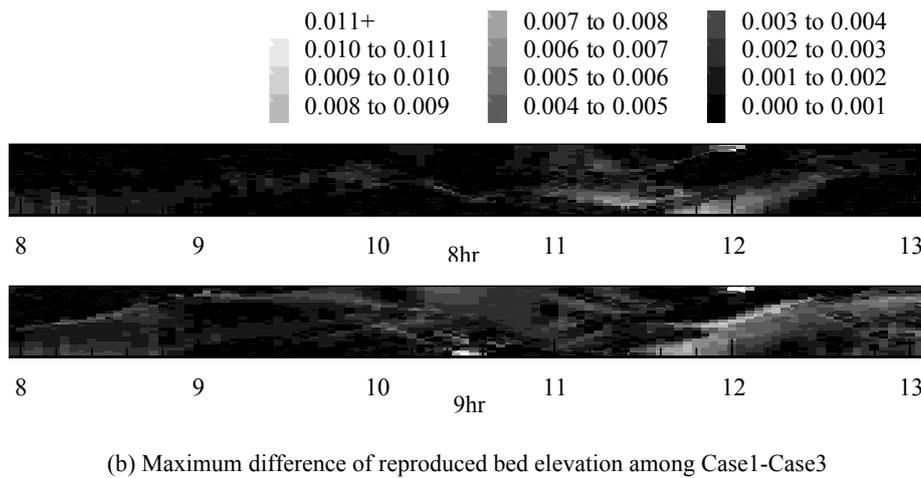
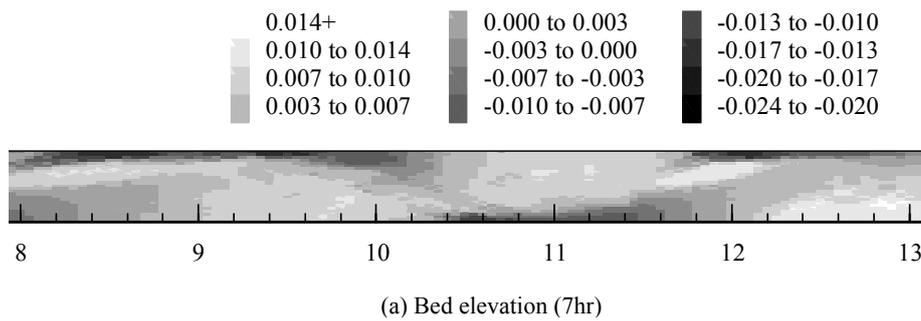


Fig.4 Plane distribution of maximum difference of bed elevation among Case1~Case3. Water flows from left to right. White areas indicate large difference.

and 3 are the same as that in Case 1. There is no apparent difference in bed geometry among three cases at 8 hour, but large difference in island geometry at 9 hour among them can be observed. This result indicates that small bed disturbance can change the meso-scale bed geometry after the long time. This result also indicates that it is difficult to predict long term bed deformation of braided stream using high numerical analysis technique and initial topographical data with high accuracy. Of course, it is difficult to predict bed deformation strictly even in a short term. However, it is considered that sensitivity of flow and bed deformation against small

disturbances distributes spatially. Hence, short term prediction of bed deformation in the low sensitive area can be performed.

Case 4 and Case 5 are performed to confirm the horizontal distribution of flow and bed deformation sensitivity against small disturbances. The same bed disturbances are set on bed in Cases 4 and 5. However, the bed disturbance is set on step in Case 4 and on pool in Case 5 respectively. **Fig.2** shows that horizontal distribution of depth integrated water velocity vectors at 8 and 9 hours in Cases 4 and 5. Comparing these two cases to Case 1, the area of island at 9 hour in Case 4 is wider than that in Case 1. On the other hand, the area of island at 9 hours in

Case 5 is almost identical to that in Case 1. These results indicate that disturbance at steps develops faster and more than that at pools and affect more on channel scale flow and bed deformation. This effect depends on the scale of bed disturbance. **Fig.3** shows the horizontal distribution of depth averaged water velocity vectors at 8 and 9 hours in Case 6 which has larger initial bed disturbance. This result indicates that horizontal distribution of water velocity vectors is changed by the disturbance in pools, when the disturbance scale is enough large.

Fig.4 shows the horizontal distribution of difference between the highest bed level and the lowest bed level among Cases 1-3. Bed geometry at 7 hour is also depicted in the figure. This result indicates that the degree of reliability of the reproduced bed geometry on braided streams distributes spatially. White area indicates the difference between the highest bed level and the lowest bed level among Cases 1-3 is large and the reliability of the reproduced bed geometry is low. The figure also shows that the region in low reliability spreads from 8 hour to 9 hour. Hence, it is able to assess the reproduced bed geometry by numerical analysis using the spatial distribution diagram of reliability. Of course, occurrence frequency of bed elevation can be obtained with more calculation cases and the frequent bed level is predicted. Furthermore, it is considered that variance of frequency distribution which is the reliability of reproduced bed geometry can be obtained.

5. CONCLUSIONS

Effect of small bed disturbances on spatiotemporal change characteristics of stream geometry is discussed using numerical analysis. Furthermore, the characteristics of reproduced bed geometry on braided streams using numerical analysis are discussed. The obtained results are summarized as follows:

- (1) Small bed disturbances develop on braided streams and affect on meso-scale bed geometry after a long term calculation. This result also indicates that it is difficult to predict long term bed deformation on braided stream using high numerical analysis technique and initial topographical data with high accuracy.
- (2) Disturbance at steps develops faster and more than that at pools and affect more on flow and bed deformation.

- (3) Degree of reliability of the reproduced bed geometry on braided streams distributes spatially. Hence, if numerical analysis performed under many cases with different bed disturbances, spatial distribution of reliability of the reproduced bed geometry can be obtained and it becomes easy to assess the reproduced bed geometry.

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